MATH 4140: Assignment 1

Chapter 1

1. Show that the symmetric group S_n (where $n \geq 1$) is not a simple group unless $n = 2$.

2. Let G and H be groups, with G simple, and let $\theta : G \to H$ be a homomorphism of groups. Show that either θ is injective, or that $\mathrm{Im}(\theta) = \{1\}$.

3. Do Exercise 4(b) from Chapter 1 of the book.

Chapter 2

4. Let $V = \mathbb{R}^2$. Find an example of a linear transformation $\theta : V \to V$ for which $\mathrm{Ker}(\theta) = \mathrm{Im}(\theta)$.

5. Let $V = \mathbb{R}^2$. Let $\pi : V \to V$ be the linear transformation whose matrix with respect to the standard basis $\mathcal{B} = \{(1,0),(0,1)\}$ is

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

   Prove that π is a projection, and find the kernel and image of π.

6. Let V be a vector space over $\mathbb{R}$. Show that if λ is an eigenvalue of a projection $\theta : V \to V$, then either $\lambda = 1$ or $\lambda = 0$.

7. Let V be a vector space over $\mathbb{R}$. Show that the only invertible projection $\theta : V \to V$ is the identity map.