1. (20)

(i) Calculate $\phi(11110000)$.

(ii) Let $N = 2^{35} - 1$. Show that $N$ is not prime by finding a factor $c$ of $N$ other than 1 and $N$, justifying your answer briefly. (Hint: you do not need a calculator. For 5 bonus points: find three factors of $N$ other than 1, $N$, and the factor $c$ already found.)

2. (20) Find a simultaneous solution $x$ to the congruences

$$x \equiv 17 \pmod{65}$$
$$x \equiv 19 \pmod{12}.$$

3. (20) Find the number $a$ with $0 \leq a < 33$ such that

$$7^{17} \equiv a \pmod{33}.$$

Using Euler's formula, or otherwise, deduce the value of

$$7^{357} (\text{mod } 33),$$

justifying your answer.

4. (20) Solve the congruence $x^7 \equiv 2 \pmod{15}$.

5. (20) True, False or Open. Mark with a "T" (true), an "F," (false) or an "O" (open, i.e., nobody knows) and provide a brief explanation (a couple of lines), for each part.

(i) _______________Euler's $\phi$-function satisfies $\phi(mn) = \phi(m)\phi(n)$ whenever $m$ and $n$ are natural numbers.

(ii) _______________If $n > 1$ is odd and $2^{n-1} \not\equiv 1 (\text{mod } n)$ then $n$ is not prime.

(iii) _______________There are infinitely many primes $p$ such that $p-2$ is also prime.

(iv) ______________ There are infinitely many primes congruent to 9 modulo 15.

(v) ______________ All perfect numbers are even.

(vi) ______________ If $k$, $b$ and $m$ are natural numbers then it is possible for the congruence

$$x^k \equiv b \pmod{m}$$

not to have any solutions.

**Name:** ______________________________

University of Colorado

Mathematics 3110: Second In-Class Exam

March 17, 2004

| Problem | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| Total | 100 | |