Math 3001 Analysis 1
Homework Set 2

Spring 2021

Course Instructor: Dr. Markus Pflaum

Contact Info: Office: Math 255, Telephone: 2-7717, e-mail: markus.pflaum@colorado.edu.

Problem 1: For which natural numbers does the inequality $2^n < n!$ hold true? (4P)

Problem 2: Let a, b be real numbers such that $0 < a < b$. Prove by induction that $a^n < b^n$ for all positive integers n. (3P)

Problem 3: Let a, b be real numbers such that $0 < a < b$. Prove that $\sqrt{a} < \sqrt{b}$. (3P)

Problem 4: Show that for all $x, y \in \mathbb{R}$

$$\max\{x, y\} = \frac{1}{2}(x + y + |x - y|) \quad \text{and} \quad \min\{x, y\} = \frac{1}{2}(x + y - |x - y|)$$

(4P)

Problem 5: Using the binomial formula show the following inequality for all natural $n \geq 2$ and real $x \geq 0$:

$$(1 + x)^n \geq \frac{n^2}{4}x^2$$

(3P)

Problem 6: Show that the supremum of the set $D = \{\frac{n^2}{2^n} \mid n \in \mathbb{N}\}$ is $\frac{9}{8}$.
Hint: First prove and then use the inequality $n^2 \leq 2^n$ for $n \geq 4$. (3P)