

Math 3001 Analysis 1
Homework Set 1

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Problem 1: Let M, N, L be sets.

a) Prove the following rule of de Morgan:

$$M \setminus (N \cup L) = (M \setminus N) \cap (M \setminus L).$$

b) Prove the following distributivity law:

$$M \cap (N \cup L) = (M \cap N) \cup (M \cap L).$$

(4P)

Problem 2: Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be mappings. Prove the following claims:

a) If f and g are injective, then $g \circ f$ is injective as well.

b) If f and g are surjective, then $g \circ f$ is surjective, too.

(4P)

Problem 3:

a) Let $f : X \rightarrow Y$ be a mapping, and $A, B \subset Y$. Show that then

$$\begin{aligned} f^{-1}(A \cap B) &= f^{-1}(A) \cap f^{-1}(B) \\ f^{-1}(A \cup B) &= f^{-1}(A) \cup f^{-1}(B). \end{aligned}$$

b) Determine, whether the following equalities are true for subsets $C, D \subset X$:

$$\begin{aligned} f(C \cap D) &= f(C) \cap f(D) \\ f(C \cup D) &= f(C) \cup f(D). \end{aligned}$$

(6P)

Problem 4: Prove the following statements for all positive integers n and real numbers $q \neq 1$:

a) $\sum_{k=1}^n k^3 = \left(\sum_{k=1}^n k \right)^2 = \frac{n^2(n+1)^2}{4},$

b) $\sum_{k=0}^n q^k = \frac{1-q^{n+1}}{1-q}$ (finite geometric series).

(6P)

Problem 5: Provide the definition of an equivalence relation

(2P).