Hilbert Spaces and the Mathematics of Quantum (Information) Theory

Augmester 2025

listed under

MATH 4810/5810, Special Topics in Mathematics

Course Instructor: Dr. Markus Pflaum

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Lecture Hourse: MTWThF 9:00 a.m. – 12:00 p.m., August 4 – 20, 2025

Venue: ECCR 105

Targeted Audience: The course is intended for upper-division undergraduate and graduate students in mathematics, physics, chemistry, computer science or engineering with an interdisciplinary interest.

Basic knowledge of Linear Algebra and Analysis is recommended.

Course Homepage: http://math.colorado.edu/courses/HilbertSpaces

Course Contents: 1925 is widely regarded as the birth year of modern quantum mechanics. The course will provide an introduction to the mathematical foundations of quantum mechanics as invented 100 years ago. On the mathematical side, the notions of a hermitian inner product, Hilbert space, bounded linear operator, Hilbert basis and Fourier expansion, selfadjointness and the spectrum of a linear operator will be explained. These concepts will then be used to describe the foundational work by Heisenberg, Schrödinger, von Neumann and others by laying out the axiomes of quantum mechanics and by application of the so-called spectral theorem. In addition, basic concepts of classical mathematical communication theory à la Shannon will be introduced and their quantum counterparts like e.g. the von Neumann entropy will be explained.

Course Projects: Each student has to write a short paper (around 5 pages) on a particular topic from the theory of Hilbert Spaces or work on an extended homework problem. In addition, a short in class presentation on the course paper or homework has to be given. The papers are due August 20, 2025. A selection of possible topics will be provided on the course page, but you can propose your own project theme.

Course Grading: Your grade will be determined from the homework or the course paper and the corresponding presentation.

Course Literature: The course will be based mainly on textbooks which are freely available for CU students as eBooks through `http://libraries.colorado.edu` or as online lecture notes under an appropriate open document license.

- Hall, *Quantum Theory for Mathematicians*, Springer Verlag
- Moretti, *Spectral Theory and Quantum Mechanics With an Introduction to the Algebraic Formulation*, Springer Verlag
- Teschl, *Mathematical Methods in Quantum Mechanics With Applications to Schrödinger Operators*, American Mathematical Society
- Wilde, *Quantum Information Theory*, 2nd edition, Cambridge University Press, online version available under `https://arxiv.org/abs/1106.1445`

The following online textbooks can be used to recall the prerequisites from Linear Algebra and Analysis and are available under an open document license.

- Hefferon, *Linear Algebra*
- Trench, *Introduction to Real Analysis*

Credits: The standard number of credit hours for this course is 3.