Series Convergence Tests (Modified from various sources)

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to show convergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum $= \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} b_n - b_{n+1}$	$\lim_{n\to\infty} b_n = L$		Sum $= b_1 - L$
p**-series**	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 \leq a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Note: Should not be used to justify divergence. Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Integral f is continuous, positive, and decreasing	$\sum_{n=1}^{\infty} a_n$ $f(n) = a_n \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $\int_{N+1}^{\infty} f(x)\,dx < R_N < \int_N^{\infty} f(x)\,dx$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Direct Comparison	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	$a_n \geq 0, b_n > 0$ $0 \leq \lim_{n\to\infty} \frac{a_n}{b_n} = L < \infty$ and $\sum_{n=1}^{\infty} b_n$ converges	$a_n \geq 0, b_n > 0$ $0 < \lim_{n\to\infty} \frac{a_n}{b_n}$ and $\sum_{n=1}^{\infty} b_n$ diverges	