6.6

Work

Work = Force × Distance

Ex work done to lift a 5 kg mass 7m up is $(5\,\text{kg}\cdot 9.8\,\text{m}/\text{s}^2)(7\,\text{m})$

$$= (35\cdot 9.8)\ \text{kg}\,\text{m}^2/\text{s}^2$$
$$= (35\cdot 9.8)\ \text{N}\,\text{m}$$
$$= (35\cdot 9.8)\ \text{J}$$

Remark Work $= \int_0^7 5\cdot 9.8\, dx$

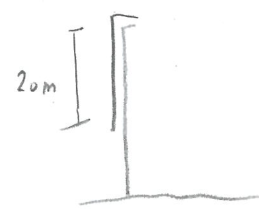
Ex (Requiring integrals)

work required to lift a 20m cable (density = 100kg/m) up the side of a building

$\rho = 100\,\text{kg/m}$

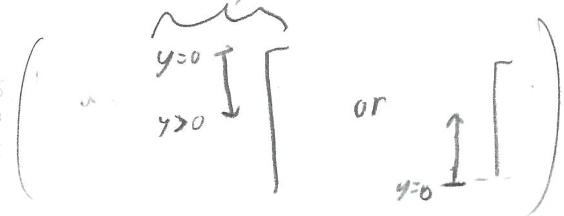
↑

"rho"



using this setup

1) Select an origin and orientation for the y-axis



2) Compute $\Delta W = g\cdot \text{mass}(\,[\,]\ [\Delta y]\,)\cdot(y)$

$$= g\cdot(\rho\cdot\Delta y)\cdot y$$
$$= 9.8\cdot 100\ \Delta y\cdot y$$
$$= 480\, y\, \Delta y$$

So $W = \int_{y=0}^{y=100} dW = \int_0^{100} 980\, y\, dy$

$$= 490\, y^2 \Big|_0^{100}$$
$$= 4900000\ \text{J}$$

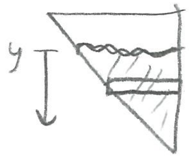
Hydrostatic Pressure/Force

$P = \text{pressure} = \rho \cdot g \cdot d$

ρ density, gravitational constant, depth

$\text{Force} = \text{Pressure} \cdot \text{Area}$

Partial Ex



Find Force on glass

1) width: $w(y)$ (The hard part)

2) $\Delta A = w(y)\,\Delta y$
   $P = \rho \cdot g \cdot y$

3) $\Delta F = P \cdot \Delta A = \rho \cdot g \cdot y \cdot w(y)\,\Delta y$

4) So $F = \int_{y=0}^{y=\text{bottom}} \rho \cdot g \cdot y \cdot w(y)\,dy$

Mass

Discrete

Total mass (TM) $= m_1 + m_2 + \cdots + m_n$

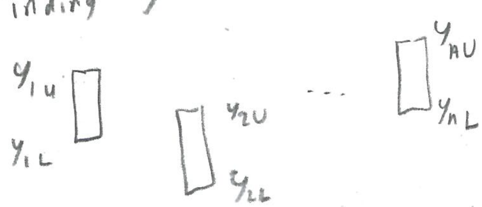
$$= \sum_{k=1}^{n} m_k$$

(First) moment $\sum_{k=1}^{n} x_k m_k$ (x_i = dist from y-axis)

second moment $\sum_{k=1}^{n} x_k^2 m_i$

<u>center of mass</u> $\bar{x} = \sum x_k \frac{m_k}{TM} = \frac{\sum x_k m_k}{\sum m_k}$

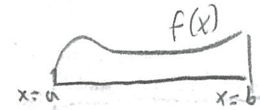
Finding $\bar{y}$



$$\bar{y} = \sum_{k=1}^{n} \left(\frac{y_{kU} + y_{kL}}{2} \right) \frac{m_k}{TM}$$

$$= \frac{\sum_{k=1}^{n} \left(\frac{y_{kU} + y_{kL}}{2} \right)}{\sum_{k=1}^{n} m_k}$$

Continuous



Total Mass (TM) $= \int_a^b (\rho f(x))\, dx$

density

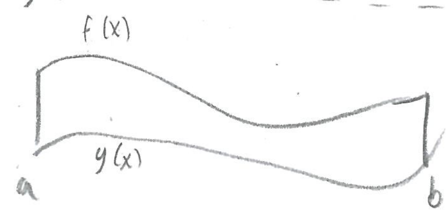
(First) moment $\int_a^b x(\rho f(x))\, dx$

Second moment $\int_a^b x^2 (\rho f(x))\, dx$

<u>Center of mass</u>

$$\bar{x} = \int_a^b x \frac{(\rho f(x))}{TM} dx = \frac{\int_a^b x(\rho f(x))\, dx}{\int_a^b (\rho f(x))\, dx} = \boxed{\frac{\int_a^b x f(x)\, dx}{\text{Area}}}$$

Finding $\bar{y}$



$$\bar{y} = \int_a^b \frac{(f(x) + g(x))}{2} \frac{(\rho(f(x) - g(x))}{TM} dx$$

$$= \frac{\int_a^b \rho \frac{(f(x))^2 - (g(x))^2}{2} dx}{\int_a^b \rho (f(x) - g(x))\, dx} = \frac{\int_a^b \frac{1}{2} \left[(f(x))^2 - (g(x))^2 \right]}{\text{Area}}$$