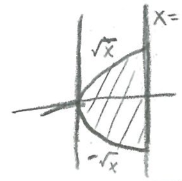
Volumes of Solids

(By cross section 6.2)

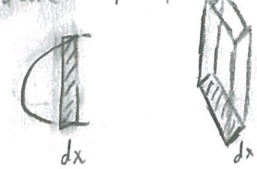
Description Base region in x-y plane

Ex R is bounded by $x=1$, $y=\pm\sqrt{x}$



Cross section (defined perpendicular to some axis)

Squares perpendicular to x-axis



{identify shade regions

Solution: Find dV (infinitessimal change in volume)

$$dV = A(x) \cdot dx = (\underbrace{S(x)}_{\text{side length}})^2 dx$$

$$= (\sqrt{x} - (-\sqrt{x}))^2 dx$$

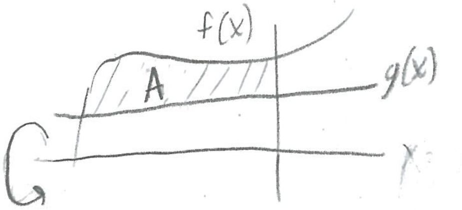
Integrate

$$\int_{x=0}^{x=1} dV = \int_0^1 (\sqrt{x} - (-\sqrt{x}))^2 dx$$

$$= \int_0^1 4x \, dx = 2x \Big|_0^1 = 2$$

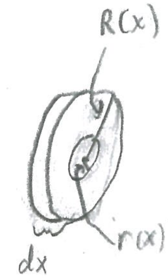
Volumes of Solids (of revolution)

(6.1 washer Method)



Rotate region A about axis

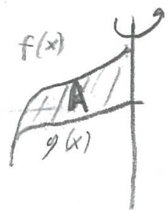
cross section



$$dV = \left[\pi\left(R(x)\right)^2 - \pi\left(r(x)\right)^2\right] dx$$

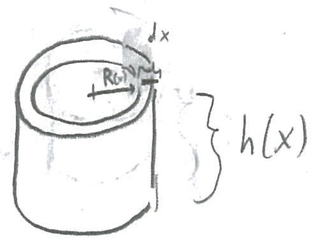
(Area of washer $= \pi R^2 - \pi r^2$)

(6.3 shell Method)



Rotate region A about axis

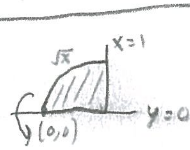
shell



$$dV = 2\pi\, R(x)\, h(x)\, dx$$

(volume of cylindrical shell of width dx $= 2\pi R h\, dx$)

Ex Rotate



about $y=0$

← solid of revolution

washer: $V = \int_{x=0}^{x=1} dV = \int_0^1 \pi\left((\sqrt{x}-0)^2 - (0-0)^2\right) dx = \pi\int_0^1 x\, dx = \frac{\pi}{2}$

Match as expected

shell: $V = \int_{y=0}^{y=1} dV = \int_0^1 2\pi(y-0)(y^2-0)\, dy = \int_0^1 2\pi y^3\, dy = \frac{\pi}{2}$