5.5

Substitution

Undoing the Chain rule,

Let $\frac{d}{dx}\left[f(u(x))\right] = f'(u(x))\,u'(x)$

So $\displaystyle\int_{x=a}^{x=b} f'(\underbrace{u(x)}_{u})\,\underbrace{u'(x)\,dx}_{du} = \int_{\substack{x=a\\u=u(a)}}^{\substack{x=b\\u=u(b)}} f'(u)\,du = f(u)\Big|_{\substack{u=u(a)\\x=a}}^{\substack{x=b\\u=u(b)}} = f(u(a))-f(u($

Recall $du = u'(x)\,dx$ when $u = u(x)$.

Ex $I = \displaystyle\int_0^{\sqrt{\pi}/2} x\sin(2x^2)\,dx$

Let $u = 2x^2$, then $du = 4x\,dx$ or $dx = \frac{du}{4x}$

So $I = \displaystyle\int_{2\cdot 0^2}^{2\left(\sqrt{\frac{\pi}{2}}\right)^2} \frac{x}{4x}\sin(u)\,du = \int_0^{\pi}\frac{1}{4}\sin(u)\,du = \frac{1}{4}\left(-\cos(u)\right)\Big|_0^{\pi} = \frac{1}{2}$

5.6

Integration by parts

Understanding the formula $\boxed{\int u\,dv = uv - \int v\,du}$

The product rule says $\frac{d}{dx}u(x)v(x) = v(x)\,u'(x) + u(x)\,v'(x)$

So $\displaystyle\int \underbrace{v(x)}_{v}\,\underbrace{u'(x)\,dx}_{du} + \int \underbrace{u(x)}_{u}\,\underbrace{v'(x)\,dx}_{dv} = \int \frac{d}{dx}\left[u(x)v(x)\right]dx = \underbrace{u(x)v}_{uv}$

So $\int v\,du + \int u\,dv = uv$

Ex $I = \int x e^x\,dx$

Let $u = x$, $dv = e^x\,dx$

$du = dx$, $v = e^x$

So $I = xe^x - \int e^x\,dx = xe^x - e^x + C$