Sample Exponential and Logarithm Problems

1 Exponential Problems

Example 1.1 *Solve* $\left(\frac{1}{6}\right)^{-3x-2} = 36^{x+1}$.

Solution: Note that $\frac{1}{6} = 6^{-1}$ and $36 = 6^2$. Therefore the equation can be written

$$(6^{-1})^{-3x-2} = (6^2)^{x+1}$$

Using the power of a power property of exponential functions, we can multiply the exponents:

$$6^{3x+2} = 6^{2x+2}$$

But we know the exponential function 6^x is one-to-one. Therefore the exponents are equal,

$$3x + 2 = 2x + 2$$

Solving this for x gives $\boxed{x = 0}$.

Example 1.2 *Solve* $25^{-2x} = 125^{x+7}$.

Solution: Note that $25 = 5^2$ and $125 = 5^3$. Therefore the equation is

$$(5^2)^{-2x} = (5^3)^{x+7}$$

Using the power of a power property to multiply exponents gives

$$5^{-4x} = 5^{3x+21}$$

Since the exponential function 5^x is one-to-one, the exponents must be equal:

$$-4x = 3x + 21$$

Solving this for x gives $\boxed{x = -3}$.

Example 1.3 *Solve* $e^x e^2 = \dfrac{e^4}{e^{x+1}}$.

Solution: Using the product and quotient properties of exponents we can rewrite the equation as

$$\begin{aligned} e^{x+2} &= e^{4-(x+1)} \\ &= e^{4-x-1} \\ &= e^{3-x} \end{aligned}$$

Since the exponential function e^x is one-to-one, we know the exponents are equal:

$$x + 2 = 3 - x$$

Solving for x gives $\boxed{x = \dfrac{1}{2}}$.

2 Log Problems

Example 2.1 *Wite the follwing equations in exponential form:*

(a) $2 = \log_3 9$

(b) $-3 = \log_e \dfrac{1}{e^3}$

(c) $\dfrac{1}{2} = \log_{81} 9$

(d) $\log_4 16 = 2$

(e) $\log_{10} 0.0001 = -3$

Solution: Use the correspondence $\log_a y = x \iff y = a^x$:

(a) $2 = \log_3 9 \iff \boxed{9 = 3^2}$

(b) $-3 = \log_e \dfrac{1}{e^3} \iff \boxed{\dfrac{1}{e^3} = e^{-3}}$

(c) $\dfrac{1}{2} = \log_{81} 9 \iff \boxed{9 = 81^{1/2}}$

(d) $\log_4 16 = 2 \iff \boxed{16 = 4^2}$

(e) $\log_{10} 0.001 = -3 \iff \boxed{0.001 = 10^{-3}}$

Example 2.2 *Wite the follwing equations in log form:*

(a) $2^{-3} = \frac{1}{8}$

(b) $8^0 = 1$

(c) $\left(\frac{1}{7}\right)^{-2} = 49$

(d) $27^{-2/3} = \frac{1}{9}$

(e) $a^b = c$

Solution: Use the correspondence $\quad y = a^x \iff \log_a y = x$:

(a) $2^{-3} = \frac{1}{8} \iff \boxed{\log_2 \frac{1}{8} = -3}$

(b) $8^0 = 1 \iff \boxed{\log_8 1 = 0}$

(c) $\left(\frac{1}{7}\right)^{-2} = 49 \iff \boxed{\log_{\frac{1}{7}} 49 = -2}$

(d) $27^{-2/3} = \frac{1}{9} \iff \boxed{\log_{27} \frac{1}{9} = -\frac{2}{3}}$

(e) $a^b = c \iff \boxed{\log_a c = b}$

Example 2.3 *Solve* $-15 = -8\ln(3x) + 7$.

Solution: Subtract 7 from both sides and divide by -8 to get

$$\frac{11}{4} = \ln(3x)$$

Note, ln is the **natural logarithm**, which is the logarithm to the base e: $\ln y = \log_e y$. Now, the equation above means

$$\frac{11}{4} = \log_e(3x)$$

so by the correspondence $y = a^x \iff \log_a y = x$,

$$3x = e^{11/4}$$

which means

$$\boxed{x = \frac{1}{3}e^{11/4}}$$

Example 2.4 *Write the expression* $\log_6 30 - \log_6 10$ *as a single term.*

Solution: This just means use the quotient rule:

$$\log_6 30 - \log_6 10 = \log_6 \frac{30}{10} = \boxed{\log_6 3}$$

Example 2.5 *Solve* $\log x - 1 = -\log(x-9)$.

Solution: Put all logarigthms on the same side, and all numbers on the other side, so we can use the correspondence $y = a^x \iff \log_a y = x$:

$$\log x + \log(x-9) = 1$$

Use the product rule to simplify the left side,

$$\log(x(x-9)) = 1$$

Note, $\underline{\log y \text{ means the base is to be understood as } 10}$, that is we have

$$\log_{10}(x(x-9)) = 1$$

By the correspondence we know

$$x(x-9) = 10^1 = 10$$

That is,

$$x^2 - 9x - 10 = 0$$

This polynomial factors: $(x-10)(x+1) = 0$, so $x = 10$ or $x = -1$. Looking at the original equation, we see we can't use $x = -1$, because $\log(-1)$ and $\log(-1-9) = \log(-10)$ are undefined. Thus, our only solution is

$$\boxed{x = 10}$$