Formula Sheet

1 Factoring Formulas

For any real numbers a and b,

$(a+b)^2$	$= a^2+2ab+b^2$	Square of a Sum
$(a-b)^2$	$= a^2-2ab+b^2$	Square of a Difference
a^2-b^2	$= (a-b)(a+b)$	Difference of Squares
a^3-b^3	$= (a-b)(a^2+ab+b^2)$	Difference of Cubes
a^3+b^3	$= (a+b)(a^2-ab+b^2)$	Sum of Cubes

2 Exponentiation Rules

For any real numbers a and b, and any rational numbers $\frac{p}{q}$ and $\frac{r}{s}$,

$a^{p/q}a^{r/s}$	$= a^{p/q+r/s}$	Product Rule
	$= a^{\frac{ps+qr}{qs}}$	
$\frac{a^{p/q}}{a^{r/s}}$	$= a^{p/q-r/s}$	Quotient Rule
	$= a^{\frac{ps-qr}{qs}}$	
$(a^{p/q})^{r/s}$	$= a^{pr/qs}$	Power of a Power Rule
$(ab)^{p/q}$	$= a^{p/q}b^{p/q}$	Power of a Product Rule
$\left(\frac{a}{b}\right)^{p/q}$	$= \frac{a^{p/q}}{b^{p/q}}$	Power of a Quotient Rule
a^0	$= 1$	Zero Exponent
$a^{-p/q}$	$= \frac{1}{a^{p/q}}$	Negative Exponents
$\frac{1}{a^{-p/q}}$	$= a^{p/q}$	Negative Exponents

Remember, there are different notations:

$$\sqrt[q]{a} = a^{1/q}$$
$$\sqrt[q]{a^p} = a^{p/q} = (a^{1/q})^p$$

3 Quadratic Formula

Finally, the **quadratic formula**: if a, b and c are real numbers, then the quadratic polynomial equation

$$ax^2 + bx + c = 0 \tag{3.1}$$

has (either one or two) solutions

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \tag{3.2}$$

4 Points and Lines

Given two points in the plane,

$$P = (x_1, y_1), \quad Q = (x_2, y_2)$$

you can obtain the following information:

1. The **distance** between them, $d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. The coordinates of the **midpoint** between them, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
3. The **slope** of the line through them, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$.

Lines can be represented in three different ways:

Standard Form	$ax + by = c$
Slope-Intercept Form	$y = mx + b$
Point-Slope Form	$y - y_1 = m(x - x_1)$

where a, b, c are real numbers, m is the slope, b (different from the standard form b) is the y-intercept, and (x_1, y_1) is *any* fixed point on the line.

5 Circles

A **circle**, sometimes denoted $\odot$, is by definition the set of all points $X := (x, y)$ a fixed distance r, called the **radius**, from another given point $C = (h, k)$, called the **center** of the circle,

$$\odot \overset{\text{def}}{=} \{X \mid d(X, C) = r\} \tag{5.1}$$

Using the distance formula and the square root property, $d(X, C) = r \iff d(X, C)^2 = r^2$, we see that this is precisely

$$\odot \overset{\text{def}}{=} \{(x, y) \mid (x - h)^2 + (y - k)^2 = r^2\} \tag{5.2}$$

which gives the familiar equation for a circle.

6 Functions

If A and B are subsets of the real numbers $\mathbb{R}$ and $f : A \to B$ is a function, then the **average rate of change** of f as x varies between x_1 and x_2 is the quotient

$$\text{average rate of change} \;=\; \frac{\Delta y}{\Delta x} \;=\; \frac{y_2 - y_1}{x_2 - x_1} \;=\; \frac{f(x_2) - f(x_1)}{x_2 - x_1} \tag{6.1}$$

It's a *linear approximation* of the behavior of f between the points x_1 and x_2.

7 Quadratic Functions

The **quadratic function** (aka the parabola function or the square function)

$$f(x) = ax^2 + bx + c \tag{7.1}$$

can always be written in the form

$$f(x) = a(x-h)^2 + k \tag{7.2}$$

where $V = (h,k)$ is the coordinate of the **vertex** of the parabola, and further

$$V = (h,k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \tag{7.3}$$

That is $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$.

8 Polynomial Division

Here are the theorems you need to know:

Theorem 8.1 (Division Algorithm) *Let $p(x)$ and $d(x)$ be any two nonzero real polynomials. There there exist unique polynomials $q(x)$ and $r(x)$ such that*

$$\begin{array}{c} p(x) = d(x)q(x) + r(x) \\ \text{or} \\ \dfrac{p(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)} \end{array} \qquad \text{where} \qquad 0 \le \deg(r(x)) < \deg(d(x))$$

Here $p(x)$ is called the **dividend**, *$d(x)$ the* **divisor**, *$q(x)$ the* **quotient**, *and $r(x)$ the* **remainder**. ■

Theorem 8.2 (Rational Zeros Theorem) *Let $f(x) = a_n x^2 + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a real polynomial with integer coefficients a_i (that is $a_i \in \mathbb{Z}$). If a rational number p/q is a root, or zero, of $f(x)$, then*

$$p \;\; \text{divides} \;\; a_0 \qquad \text{and} \qquad q \;\; \text{divides} \;\; a_n$$ ■

Theorem 8.3 (Intermediate Value Theorem) *Let $f(x)$ be a real polynomial. If there are real numbers $a < b$ such that $f(a)$ and $f(b)$ have opposite signs, i.e. one of the following holds*

$$f(a) < 0 < f(b)$$
$$f(a) > 0 > f(b)$$

then there is at least one number c, $a < c < b$, such that $f(c) = 0$. That is, $f(x)$ has a root in the interval (a, b). ■

Theorem 8.4 (Remainder Theorem) *If a real polynomial $p(x)$ is divided by $(x - c)$ with the result that*

$$p(x) = (x - c)q(x) + r$$

(r is a number, i.e. a degree 0 polynomial, by the division algorithm mentioned above), then

$$r = p(c)$$ ■

9 Exponential and Logarithmic Functions

First, the all important correspondence

$$y = a^x \iff \log_a(y) = x \tag{9.1}$$

which is merely a statement that a^x and $\log_a(y)$ are inverses of each other.

Then, we have the rules these functions obey: For all real numbers x and y

$$a^{x+y} = a^x a^y \tag{9.2}$$
$$a^{x-y} = \frac{a^x}{a^y} \tag{9.3}$$
$$a^0 = 1 \tag{9.4}$$

and for all *positive* real numbers M and N

$$\log_a(MN) = \log_a(M) + \log_a(N) \tag{9.5}$$
$$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N) \tag{9.6}$$
$$\log_a(1) = 0 \tag{9.7}$$
$$\log_a(M^N) = N\log_a(M) \tag{9.8}$$