1. The sides of a square are all increasing uniformly at a rate of 3 inches/minute. At what rate is the area of the square increasing when the side length is 10 inches?

   Let x be the side length of the square. We have $A = x^2$ and thus

   $$\frac{dA}{dt} = 2x\frac{dx}{dt}.$$

   We plug in to get

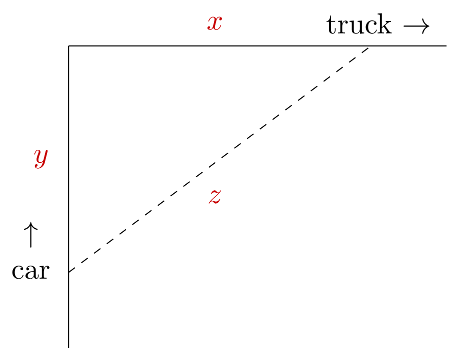
   $$\frac{dA}{dt} = 2\,(10 \text{ inches})\left(3\,\frac{\text{inches}}{\text{minute}}\right) = 60\,\text{in}^2/\text{min}.$$

2. A particle moves along the graph of $y = \tan(x)$. Its velocity in the x-direction (dx/dt) is 5 units per minute. When $x = \frac{\pi}{4}$, what is its velocity in the y-direction (dy/dt)?

   $y = \tan(x)$ and so $\frac{dy}{dt} = \sec^2(x)\frac{dx}{dt}$

   $\frac{dy}{dt} = \sec^2\left(\frac{\pi}{4}\right) \cdot (5 \text{ units/min}) = \left(\sqrt{2}\right)^2 \cdot (5 \text{ units/min}) = 10 \text{ units/min}.$

3. A car is traveling north toward an intersection at a rate of 60 mph while a truck is traveling east away from the intersection at a rate of 50 mph. Find the rate of change of the distance between the car and truck when the car is 3 miles south of the intersection and the truck is 4 miles east of the intersection.



$\frac{dx}{dt} = 50$ miles/hr $\frac{dy}{dt} = -60$ miles/hr

$x^2 + y^2 = z^2$

When $x = 4$ and $y = 3$, we have $z = \sqrt{9 + 16} = 5$ miles.

Differentiating the equation above gives

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{(4 \text{ mi})(50 \text{ mi/hr}) + (3 \text{ mi})(-60 \text{ mi/hr})}{5 \text{ mi}} = 4 \text{ miles/hr}.$$

The distance between them is increasing at 4 miles/hr.

4. A rectangle of length ℓ and width w has a constant area of 1200 in^2. The side lengths are changing while keeping the area the same. Suppose that at a particular instant the length is increasing at 6 in/min and the width is decreasing at 2 in/min.

   (a) Find the dimensions of the rectangle at this instant.

   $1200 = \ell w$ $\qquad 0 = \frac{d\ell}{dt}w + \ell\frac{dw}{dt}$

   $\frac{d\ell}{dt} = 6$ in/min $\qquad \frac{dw}{dt} = -2$ in/min

   $1200/w = \ell$ $\qquad 0 = 6w - 2\ell$

   $0 = 6w - 2(1200/w)$
   $6w^2 = 2400$
   $w^2 = 400$
   $w = 20$ inches
   $\ell = 1200/20 = 60$ inches

   (b) At this same instant, is the length of the *diagonal* increasing or decreasing? At what rate?

   $D^2 = \ell^2 + w^2$ $\qquad 2D\frac{dD}{dt} = 2\ell\frac{d\ell}{dt} + 2w\frac{dw}{dt}$

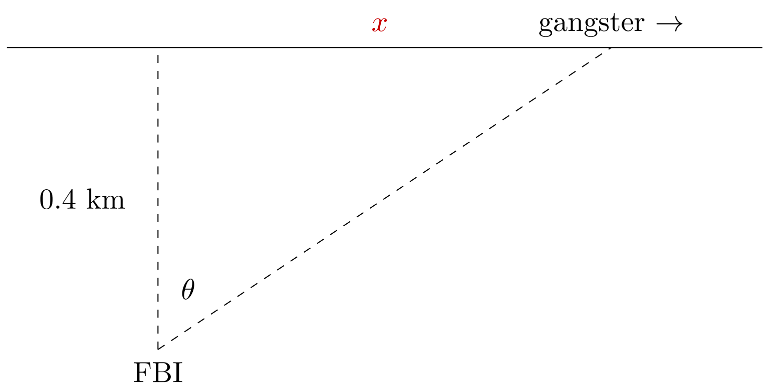
   $$\frac{dD}{dt} = \frac{\ell\frac{d\ell}{dt} + w\frac{dw}{dt}}{D}$$

   When $\ell = 60$ and $w = 20$, $D = \sqrt{60^2 + 20^2} = \sqrt{3600 + 400} = \sqrt{4000} = 20\sqrt{10}$ inches.

   $$\frac{dD}{dt} = \frac{(60 \text{ in})(6 \text{ in/min}) + (20 \text{ in})(-2 \text{ in/min})}{20\sqrt{10} \text{ in}} = \frac{320}{20\sqrt{10}} \text{ in/min} \approx 5.06 \text{ in/min}.$$

   The length of the diagonal is increasing at approximately 5.06 inches per minute.

5. An FBI agent with a powerful spyglass is located in a boat anchored 0.4 km offshore. A gangster under surveillance is walking along the shore. Assuming the shoreline is straight and that the gangster is walking at the rate of 2 km/hr, how fast must the FBI agent rotate the spyglass to track the gangster when the gangster is 1 km from the point on the shore nearest to the boat? (In other words, find $d\theta/dt$.)



$\tan\theta = \dfrac{x}{0.4}$ $\qquad\qquad$ $\sec^2\theta\,\dfrac{d\theta}{dt} = \dfrac{1}{0.4}\dfrac{dx}{dt} = 2.5\dfrac{dx}{dt}$

$\dfrac{dx}{dt} = 2 \text{ km/hr}$

When $x = 1$, we have $\tan\theta = 1/0.4$ and $\theta = \arctan(1/0.4) = \arctan(2.5)$

Useful trig identity: $1 + \tan^2 * = \sec^2 *$ for any value of $*$.

$$\frac{d\theta}{dt} = \frac{2.5\frac{dx}{dt}}{\sec^2\theta} = \frac{2.5(2 \text{ km/hr})}{\sec^2(\arctan(2.5))}$$
$$= \frac{5}{1+\tan^2(\arctan(2.5))} = \frac{5}{1+(2.5)^2} = \frac{5}{7.25} = \frac{20}{29} \text{ radians/hr.}$$