Review 4 - Short Answers

1. $\operatorname{div}\vec{F} = 4z$ and the sphere is symmetric over the xy-plane, so the flux is 0 by the Divergence theorem.

2. $\displaystyle\iint_\sigma \vec{F}\cdot d\vec{S} = \int_0^1\int_1^3 \langle 5t, s^2, 0\rangle \cdot \langle 10, -10s, 4st\rangle \; dt\, ds = 195.$

3. $\displaystyle\oint_C \vec{F}\cdot d\vec{r} \overset{\text{Stokes'}}{=} \iint_\sigma \langle 2, -3, -8\rangle \cdot \langle 1, 0, 0\rangle \; dS = 128\pi.$

4. $\displaystyle\iint_\sigma \vec{F}\cdot d\vec{S} \overset{\text{Divergence}}{=} -\left(\iiint 0\, dV - \iint_{x^2+y^2\leq 25} \langle 0, 0, \cos(x^2+y^2)\rangle \cdot \langle 0, 0, -1\rangle \; dA\right) = -\pi\sin(25).$

5. $\displaystyle\iint_\sigma \vec{F}\cdot d\vec{S} = \int_0^2\int_0^3 \langle x-y, x+y, 3x\rangle \cdot \langle -1, -1, 1\rangle \; dy\, dx = 6.$

6. $\displaystyle\iint_\sigma \vec{F}\cdot d\vec{S} = \int_0^\pi\int_1^3 \left\langle \frac{-2}{a\cos\theta}, \frac{2}{a\sin\theta}, 0\right\rangle \cdot \langle -2a^2\cos\theta\cos(a^2), -2a^2\sin\theta\cos(a^2), a\rangle \; da\, d\theta = 0.$

7. $\displaystyle\oint_C \vec{F}\cdot d\vec{r} \overset{\text{Stokes'}}{=} \int_0^5\int_0^{\frac{3}{5}z} \langle -1, -1, -1\rangle \cdot \langle -1, 0, 0\rangle \; dy\, dz = \frac{15}{2}.$

8. $\operatorname{div}\vec{F} = 1$ so, by the Divergence theorem, the flux is equal to $(-1)\cdot$ volume (from orientation). Thus, the flux is -12.