Math 2400 Midterm Review 2

1. Consider the surface S determined by the equation $2x^2 + 3y^2 + z^2 = 20$.

   (a) Verify that the point $P = (2, 1, 3)$ is a point on S and find the equation of the tangent plane to S at this point.

   (b) The above equation defines z implicitly as a function of x and y, $z = f(x, y)$. Find the local linear approximation for $f(x, y)$ at $(2, 1)$.

   (c) Approximate the value of z corresponding to $x = 1.97$ and $y = 1.12$.

2. Let $w = f(\rho)$, where $\rho = \sqrt{x^2 + y^2 + z^2}$. Show that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 = \left(\frac{\partial f}{\partial \rho}\right)^2.$$

3. An object's specific gravity S can be found using the formula

$$S = \frac{A}{A - W}$$

   where A is the number of pounds the object weighs in air and W is the number of pounds the object weighs in water. The weight in air A is measured to be 10 ± 0.1 lbs, and the weight in water W is measured to be 8 ± 0.08 lbs. Use differentials to estimate the maximum possible error in calculating specific gravity.

4. Consider the function

$$f(x, y) = \begin{cases} \dfrac{x^3 y^2}{x^4 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

   (a) Is $f(x, y)$ continuous everywhere? If not, where is it not continuous.

(b) What are $f_x(0,0)$ and $f_y(0,0)$?

(c) Are $f_x(x,y)$ and $f_y(x,y)$ continuous?

(d) Is $f(x,y)$ differentiable everywhere? If not, where is it not differentiable.

5. An airline limits the size of luggage that a passenger can carry by requiring that the sum of the length, width, and height be at most 135 cm. Find the largest volume of luggage that a passenger is allowed to carry.

6. (a) Find the point on the paraboloid $z = x^2 + y^2$ that is closest to the point $(3, 4, -1)$.

   (b) Find a vector normal to the paraboloid $z = x^2 + y^2$ at the point found in (a).

   (c) Find the tangent plane of the paraboloid $z = x^2 + y^2$ at the point found in (a).

   (d) Find the vector from the point on the paraboloid you found in (a) to $(3, 4, -1)$. What is the cross product of this vector with the vector you found in part (b)?

   (e) Suppose you are standing on the paraboloid at the point you found in (a). If you want to stay on the surface and increase z the fastest, in what 3D direction do you want to travel?

7. (a) What is the second order Taylor polynomial $p_2(x)$ for $f(x) = e^x$ at $x = 0$?

   (b) What is the second order Taylor polynomial $P_2(x,y)$ for $f(x,y) = e^{2x+3y}$ at $(0,0)$?

   (c) With your answer $p_2(x)$ from part (a), what is $q(x,y) = p_2(2x)p_2(3y)$? How does $q(x,y)$ compare to your answer in part (b)?