Math 2400 Midterm Review 1

1. Consider the points $A = (4, -2, 3)$, $B = (8, -7, 6)$, and $C = (6, -4, 6)$.

   (a) Find the area of the parallelogram spanned by the vectors $\vec{AB}$ and $\vec{AC}$.

   (b) Find an equation of the plane containing the points A, B, C.

   (c) Find an equation of the sphere centered at the origin and tangent to the plane containing the points A, B, C.

2. Are the planes $\Pi_1 \ : \ x + z = 1$ and $\Pi_2 \ : \ y + z = 1$ parallel, perpendicular or neither? If neither, find the angle between them.

3. If $P = (1, 1, 0)$, $Q = (0, 0, 1)$, and $R = (x, y, z)$, describe the set of points (x, y, z) that satisfy $\vec{PR} \cdot \vec{QR} = 0$.

4. The following statements are either true or false. If true, then say so and explain why. If false, then say so and explain why or give a counter-example to show why the statement is false.

   (a) If $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{0}$, then $\dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \ \|\vec{b}\|} = 1$.

   (b) $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \|\vec{v}\|$

   (c) The cross product of two unit vectors is a unit vector.

   (d) If $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$, then $\vec{u}$ and $\vec{v}$ are orthogonal.

   (e) $(\vec{u} + \vec{v}) \times (\vec{u} - \vec{v}) = 0$

   (f) If $\vec{v}$ and $\vec{w}$ are nonzero nonparallel vectors of the same length, then $\vec{v} + \vec{w}$ and $\vec{v} - \vec{w}$ are perpendicular.