Math 2400 Section 003 – Calculus III – Spring 2012

Review for Midterm 3 – Part 2 *Vector Fields*

1. Suppose $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ is a constant vector. Which of the following are vector fields? Explain.
   (a) $\vec{r} + \vec{a}$
   (b) $\vec{r} \cdot \vec{a}$
   (c) $x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$
   (d) $x^2 + y^2 + z^2$

2. A particle passes through the point $P = (5, 4, 3)$ at time $t = 7$, moving with constant velocity $\vec{v} = 3\vec{i} + \vec{j} + 2\vec{k}$. Find equations for its position at time t.

3. A stone is swung around on a strong at a constant speed with period 2π seconds in a horizontal circle centered at the point $(0, 0, 8)$. When $t = 0$, the stone is at the point $(0, 5, 8)$; it travels clockwise when viewed from above. When the stone is at the point $(5, 0, 8)$, the string breaks and it moves under gravity.
   (a) Parameterize the stone's circular trajectory.
   (b) Find the velocity and acceleration of the stone at the moment before the string breaks.
   (c) Write, but do not solve, the differential equations (with initial conditions) satisfied by the coordinates x, y, z giving the position of the stone after it has left the circle.

4. The motion of a particle is given by the parametric equations
$$x = t^3 - 3t, \quad y = t^2 - 2t.$$
   Give parametric equations for the tangent line to the path of the particle at time $t = -2$.

5. Find parametric equations of the line passing through the points $(1, 2, 3)$ and $(3, 5, 7)$ and calculate the shortest distance from the line to the origin.

6. If $\vec{F} = \vec{r}/||\vec{r}||^3$, find the following quantities in terms of x, y, z, and t.
   (a) $||\vec{F}||$
   (b) $\vec{F} \cdot \vec{r}$
   (c) A unit vector parallel to $\vec{F}$ and pointing in the same direction.
   (d) A unit vector parallel to $\vec{F}$ and pointing in the opposite direction.
   (e) $\vec{F}$ if $\vec{r} = \cos(t)\vec{i} + \sin(t)\vec{j} + \vec{k}$.
   (f) $\vec{F} \cdot \vec{r}$ if $\vec{r} = \cos(t)\vec{i} + \sin(t)\vec{j} + \vec{k}$.

7. Parameterize the cone of height h with maximum height a with vertex at the origin and opening upward. Do this in two ways, giving the range of values for each parameter in each case:
   (a) Use r and θ.
   (b) Use z and θ.

8. Adapt the parameterization of the sphere to find a parameterization for the ellipsoid.
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$