Math 2400 Section 003 – Calculus III – Spring 2012

Review for Midterm 3 – Part 1 *Integration*

1. Calculate the double integral

$$\iint_R xye^y \, dA, \qquad R = \{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le 1\}.$$

2. Find the average value of f over the given integral

$$f(x,y) = x\sin(xy), \qquad R = [0,\pi/2] \times [0,1].$$

3. Evaluate the integrals by reversing the order of integration.

   (a) $\displaystyle\int_0^3 \int_{y^2}^9 y\cos(x^2)\,dx\,dy$

   (b) $\displaystyle\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x\sqrt{1+\cos^2 x}\,dx\,dy$

4. Compute $\iint_D \sqrt{1-x^2-y^2}\,dA$ where D is the disk $x^2+y^2 \le 1$.

5. Evaluate the integral

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2\,dx\,dy.$$

6. A swimming pool is circular with a 40-ft diameter. The depth is constant along east-west lines and increases linearly from 2 ft at the south end to 7 ft at the north end. Find the volume of water in the pool.

7. Evaluate

$$\int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x xy\,dy\,dx + \int_1^{\sqrt{2}} \int_0^x xy\,dy\,dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy\,dy\,dx.$$

8. Find the mass and center of mass of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$, and $x+y+z=1$ with the density function $\delta(x,y,z)=y$.

9. Find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\pi/6$.

10. Find the volume of the torus defined by the equation $\rho = \sin\phi$.

11. Calculate $\displaystyle\iint_R 3x+4y\,dA$ where R is the region bounded by the lines $y=x$, $y=x-2$, $y=-2x$ and $y=3-2x$ using the transformations $x=\dfrac{1}{3}(u+v)$ and $y=\dfrac{1}{3}(v-2u)$.

12. Calculate $\displaystyle\iint_R x^2-xy+y^2\,dA$ where R is the region bounded by the ellipse $x^2-xy+y^2=2$ using the transformations $x=\sqrt{2}u-\sqrt{2/3}v$ and $y=\sqrt{2}u+\sqrt{2/3}v$.