Quiz 6

MATH 2400
July 18, 2012

1. Evaluate $\iiint\limits_G z^2\,dV$ where G is the solid above $z = \sqrt{x^2+y^2}$ and inside $x^2+y^2+z^2=9$.

$$\begin{aligned}
\iiint\limits_G z^2\,dV &= \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{3} \rho^2\cos^2\phi\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta \\
&= 2\pi\left[-\frac{1}{3}\cos^3\phi\right]_0^{\frac{\pi}{4}}\left[\frac{1}{5}\rho^5\right]_0^3 \\
&= \frac{162\pi}{5}\left(1-\frac{1}{2\sqrt{2}}\right).
\end{aligned}$$

2. Show that $\vec{r}(t) = \left\langle e^{t^2}, te^{t^2}\right\rangle$ is a flow line of $\vec{F} = \left\langle 2y, x+\frac{2y^2}{x}\right\rangle$.

$$\begin{aligned}
\vec{F}(\vec{r}(t)) &= \left\langle 2te^{t^2}, e^{t^2}+\frac{2t^2e^{2t^2}}{e^{t^2}}\right\rangle \\
&= \left\langle 2te^{t^2}, e^{t^2}+2t^2e^{t^2}\right\rangle \\
&= \left\langle \frac{d}{dt}\left[e^{t^2}\right], \frac{d}{dt}\left[te^{t^2}\right]\right\rangle \\
&= \frac{d}{dt}\left\langle e^{t^2}, te^{t^2}\right\rangle \\
&= \vec{r}\,'(t).
\end{aligned}$$

Since $\vec{F}(\vec{r}(t)) = \vec{r}\,'(t)$, then $\vec{r}(t)$ is a flow line of $\vec{F}$.

3. Evaluate $\iint\limits_R e^{\frac{x+2y}{y-3x}}\,dA$ where R is the region bounded by $y=-\frac{1}{2}x$, $y=-4x$, $y=3x+2$.

Using the change of variables $u = x+2y$, $v = y-3x$, we then have boundary curves in u,v-coordinates: $u=0$, $v=2$, $u=v$.

$$\frac{\partial(x,y)}{(u,v)} = \frac{1}{\frac{\partial(u,v)}{(x,y)}} = \frac{1}{\begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix}} = \frac{1}{7}$$

$$\iint\limits_R e^{\frac{x+2y}{y-3x}}\,dA = \int_0^2 \int_0^v e^{\frac{u}{v}} \frac{1}{7}\,du\,dv$$
$$= \frac{1}{7}\int_0^2 \left[ve^{\frac{u}{v}}\right]_0^v\,dv$$
$$= \frac{1}{7}\int_0^2 v(e-1)\,dv$$
$$= \frac{e-1}{7}\left[\frac{1}{2}v^2\right]_0^2$$
$$= \frac{2}{7}(e-1).$$

4. Parameterize the curve of intersection of $z = 13 - 2x^2$ and $z = 4x^2 + 6y^2 - 41$.

   The points on the curve will satisfy the equation

$$3 - 2x^2 = z = 4x^2 + 6y^2 - 41$$
$$54 = 6(x^2 + y^2)$$
$$9 = x^2 + y^2$$

   We can parameterize this by setting $(x, y) = (3\cos t, 3\sin t)$, $0 \le t \le 2\pi$. This would cause $z = 13 - 2x^2 = 13 - 18\cos^2 t$. Thus, the parametrization is $(x, y, z) = (3\cos t, 3\sin t, 13 - 18\cos^2 t)$, $0 \le t \le 2\pi$.