Quiz 2

MATH 2400

June 13, 2012

1. Let f be the function $f(x,y) = c + mx + ny$, where c, m, n are constants and $n \neq 0$. Show that all the contours of f are lines of slope $\dfrac{-m}{n}$.

   Solution: Choose some real number r. The contour line $f = r$ is the collection of points (x, y) such that $c + mx + ny = r$. Solving for y in terms of x, we get that $y = \frac{-m}{n}x + \frac{r-c}{n}$. This is a line with slope $\frac{-m}{n}$.

2. Find a function $f(x,y,z)$ whose level surface $f = 1$ is the graph of the function $g(x,y) = \arctan(x^2 - y^2)$.

   Solution: The graph of the function $g(x,y) = \arctan(x^2 - y^2)$ is the collection of points (x,y,z) such that $z = \arctan(x^2 - y^2)$. Consider the function $f(x,y,z) = \arctan(x^2 - y^2) - z + 1$. Now the level surface $f = 1$ is the collection of points (x,y,z) such that $\arctan(x^2 - y^2) - z + 1 = 1$, which is exactly the collection of points (x,y,z) such that $z = \arctan(x^2 - y^2)$.

3. Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^4}{2x^2+y^2}$$

Solution: Define $f(x,y) = \frac{x^2+y^4}{2x^2+y^2}$ for $(x,y) \neq 0$. Suppose we consider the value of the function for non-zero points along the x-axis. In particular, any non-zero point on the x-axis is of the form $(x,0)$ where $x \neq 0$. Evaluating the function f at these points we get $f(x,0) = \frac{x^2+0^4}{2x^2+0^2} = \frac{1}{2}$. Hence, if we approach the origin along the x-axis, the function is always $\frac{1}{2}$. Next, suppose we consider the value of the function for non-zero points along y-axis. In particular, any non-zero point on the y-axis is of the form $(0,y)$ where $y \neq 0$. Evaluating the function f at these points we get $f(0,y) = \frac{0^2+y^4}{2(0^2)+y^2} = y^2$. Hence, if we approach the origin along the y-axis, the function is y^2. Therefore, the function will be going to 0 as we approach the origin along the y-axis. Hence, the limit of the function does not exist, because there are points (x,y) arbitrarily close to 0 which have a function value of $\frac{1}{2}$ and there are points (x,y) arbitrarily close to 0 which have a function value near 0.

4. Find a unit vector in the opposite direction to $\vec{v} = 2\vec{i} - \vec{j} - \sqrt{11}\vec{k}$.

Solution: The vector $-\vec{v} = -2\vec{i} + \vec{j} + \sqrt{11}\vec{k}$ is a vector which points in the opposite direction as $\vec{v}$. To make $-\vec{v}$ unit length we need to divide the vector by its norm. Observe that $||-\vec{v}|| = \sqrt{(-2)^2 + (1)^2 + (\sqrt{11})^2} = \sqrt{16} = 4$. Hence, a unit vector in the opposite direction as $\vec{v}$ would be $\frac{-\vec{v}}{||-\vec{v}||} = \frac{-\vec{v}}{4} = \frac{-2}{4}\vec{i} + \frac{1}{4}\vec{j} + \frac{\sqrt{11}}{4}\vec{k}$.