Practice Integrals

Setup the following integrals in all reasonable coordinate systems. Compute each quantity in an appropriate coordinate system.

1. The volume the solid bounded by the planes $7x - 8y + 2z = 19$, $5x - y + z = 2$, $y = 3x + 8$, and $x = 4$.

2. The volume between $z = y^2 + 1$ and $z = 9 - 2x^2 - y^2$.

3. The mass of the solid that is bounded by the cone $z = \dfrac{1}{a}\sqrt{x^2 + y^2}$ and the plane $z = b$, and whose density is proportional to the distance from the z-axis.

4. The volume inside $x^2 + y^2 = R^2$ in the first octant, and below $z = 3x$.

5. The volume inside $x^2 + y^2 + z^2 = R^2$ and above $z = x^2 + y^2$.

6. The mass of the solid bounded below by $z = \dfrac{1}{a}\sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = R^2$, with density $\delta = 3e^{-(x^2+y^2+z^2)^{\frac{3}{2}}}$.

7. The volume inside $x^2 + y^2 + z^2 = 2z$ and below $z = 1 + \sqrt{x^2 + y^2}$.

8. The center of mass of the solid bounded by $z = \sqrt{x^2 + y^2}$ and $z = x^2 + y^2$, with density proportional to the distance from the z-axis.

9. The volume inside the cylinders $x^2 + z^2 = R^2$ and $y^2 + z^2 = R^2$.

10. The volume inside the cylinders $x^2 + z^2 = R^2$, $y^2 + z^2 = R^2$ and $x^2 + y^2 = R^2$.

11. The volume between the surfaces $xy = 1$, $xy = 2$, $xz = 1$, $xz = 3$, $yz = 1$, $yz = 4$.

12. The volume under $z = \sin(4x^2 + 9y^2)$, above the xy-plane, and inside the elliptical cylinder $4x^2 + 9y^2 = 1$.