Calculus 3 - Summer 2012

Homework #7

Due 7/23/2012

Written Problems

1. Find a point on the xy-plane that lies on the tangent planes to both of the following surfaces at the intersection point $z = 4$.

$$x = \sqrt{25 - z^2}, \qquad y = 2z - \ln(z - 3)$$

2. Consider the surface $z = f(x, y) = 100 - x^2 - 100y^2$.
   (a) Find the flow line of $\vec{\nabla} f$ through the point (a, b).
   (b) Sketch the flow lines for $(a, b) = (0, 0)$, $(10, 0)$, $(0, 1)$, $\left(6, \frac{4}{5}\right)$, and $\left(8, \frac{3}{5}\right)$.

Presentation Problems

3. Suppose you are climbing a hill approximated by the surface $z = 100 - x^2 - 100y^2$ and are starting at height 0. Suppose that wherever you start, you want to reach the top the fastest, and so, you climb along the path of steepest ascent. Where should you start in order to travel straight to the top? Which of these is fastest? Slowest? What would happen if you started elsewhere?

4. Consider the helix with parametrization $\vec{r}(t) = \langle a \cos t, a \sin t, ct \rangle$.
   (a) Find the arc length of the helix for $0 \leq t \leq T$ for $T > 0$.
   (b) Find a reparametrization $t = f(s)$ such that $||\vec{r}\,'(s)|| = \left|\left|\vec{r}\,'(f^{-1}(t))\right|\right| = 1$ for all s.
   (c) Find the arc length of the helix for $0 \leq s \leq S$ for $S > 0$.

5. Consider the sphere $x^2 + y^2 + z^2 = 1$ and the plane $z = 0$.
   (a) Parameterize the line, $\vec{\ell}_{u,v}(t)$, through $(0, 0, 1)$ and $(u, v, 0)$ for any point $(u, v, 0)$ in the plane $z = 0$.
   (b) Find the intersection point, (x, y, z), of $\vec{\ell}_{u,v}(t)$ with the sphere.
   (c) Express this intersection point in terms of u and v. That is, $(x(u, v), y(u, v), z(u, v))$.
   (d) Does this parameterize the whole sphere?