Calculus 3 - Summer 2012

Homework #3

Due 6/25/2012

Written Problems

1. Use the limit definition of the partial derivatives to find $f_x(3,2)$ and $f_y(3,2)$ for the function $f(x,y) = \dfrac{x^2}{y+1}$.

2. Let $f(x,y)$ be a function of two variables.

   (a) If $f_x(a,b)$ or $f_y(a,b)$ is non-zero, use local linearization to show that an equation of the line tangent to the contour of f at (a,b) is $f_x(a,b)(x-a) + f_y(a,b)(y-b) = 0$.

   (b) Find the slope of the tangent line if $f_y(a,b) \neq 0$.

   (c) Find an equation for the tangent line to the contour of $f(x,y) = x^2 + xy$ at $(3,4)$.

3. Let f be a differentiable function of one variable. Show that all tangent planes to the surface $z = xf\left(\dfrac{y}{x}\right)$ intersect at a common point.

Presentation Problems

4. Define $f(x,y) = \left(\displaystyle\int_3^x e^{t^2}\,dt\right) y$. Find the directional derivative of f at the point $(3,1)$ in the direction of the vector $\vec{v} = (2,3)$.

5. Let $k > 0$. Show that the volume in the first octant, bounded by the coordinate planes and *any* tangent plane of $xyz = k$, $x, y > 0$ depends only on k. *(Hint: For $a, b, c > 0$, the volume in the first octant under the plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ is $\dfrac{abc}{3!}$.)*

6. (a) Let $z = f(x,y)$ and $z = g(x,y)$ be differentiable surfaces. Show that if $\vec{\nabla} f \cdot \vec{\nabla} g = -1$ at a point of intersection, then the surfaces are perpendicular at that point.

   (b) Show that the surfaces $z = \frac{1}{2}(x^2 + y^2 - 1)$ and $z = \frac{1}{2}(1 - x^2 - y^2)$ are perpendicular at all points of intersection.