Calculus 3 - Summer 2012

Homework #2

Due 6/18/2012

Written Problems

1. Find all 2D vectors $\vec{v}$ such that $\left\|\vec{v} - \hat{i}\right\| = \sqrt{5}$ and $\left\|\vec{v} + \hat{j}\right\| = \sqrt{5}$.

2. A plane wants to fly due North while in 70 km/hr winds blowing to the Southeast. Assuming the plane can fly at 600 km/hr in calm weather, and that the plane is flying at maximal speed, what direction should the plane point? What will be its ground speed?

3. Let $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ and $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ be non-parallel, non-zero vectors lying in $z = mx + ny + c$. Let S be a parallelogram defined by $\vec{u}$ and $\vec{v}$, and R be the parallelogram that is the projection of S into the xy-plane.

   (a) Find the area of S.

   (b) Find the area of R.

   (c) Show that

$$(\text{Area of } S) = \sqrt{m^2 + n^2 + 1} \cdot (\text{Area of } R).$$

Presentation Problems

4. (a) Prove the Cauchy-Schwarz Inequality: For any two n-dimensional vectors, we have that

$$|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \, \|\vec{v}\| \, .$$

   When will this be an equality?

   (b) Use the Cauchy-Schwarz Inequality to prove the Triangular Inequality: For any two n-dimensional vectors, we have that

$$\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$$

   When will this be an equality?

5. Suppose $\vec{u}$ and $\vec{v}$ are 3-dimensional vectors such that $\vec{u} \times \vec{w} = \vec{v} \times \vec{w}$ for every 3-dimensional vector $\vec{w}$. Show that $\vec{u} = \vec{v}$.

6. Determine if each of the following equations are plausible. For those that are, either explain why it is true, or give a counterexample why it is false.

   (a) $(\vec{u} \cdot \vec{v}) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \cdot \vec{w})$

   (b) $(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w})$

   (c) $(\vec{u} \cdot \vec{v}) \times \vec{w} = \vec{u} \cdot (\vec{v} \times \vec{w})$

   (d) $(\vec{u} \times \vec{v}) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \times \vec{w})$

   (e) For unit vectors $\vec{u}$ and $\vec{v}$, $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = 0$