Quiz 20

16.7.12:

$$\begin{aligned}
\Phi &= \iint_{\sigma} \boldsymbol{F} \cdot \boldsymbol{n}\, d\boldsymbol{r} \\
&= \iiint_{G} \operatorname{div}\boldsymbol{F}\, dV \\
&= \iiint_{G} 5z\, dV \\
&= \int_0^{2\pi} \int_0^{\pi} \int_0^{a} 5\rho^3 \sin\phi \cos\phi \, d\rho\, d\phi\, d\theta \\
&= 10\pi \left[\frac{1}{2}\sin^2\phi\right]_0^{\pi} \left[\frac{1}{4}r^4\right]_0^{a} \\
&= 10\pi \left(\frac{1}{2}\right)\left(\frac{a^4}{4}\right) \\
&= \frac{5}{4}\pi a^4.
\end{aligned}$$

16.8.12:
$\operatorname{curl}\boldsymbol{F} = \langle 1,1,1\rangle \qquad C:$ boundary of the disk $x^2+y^2 \le \frac{1}{2}, \quad z = \frac{1}{\sqrt{2}}$

$$\oint_C \boldsymbol{F} \cdot d\boldsymbol{r} = \iint_{x^2+y^2\le\frac{1}{2}} \langle 1,1,1\rangle \cdot \langle 0,0,1\rangle \; dA = \iint_{x^2+y^2\le\frac{1}{2}} dA = \frac{\pi}{2}$$

HB:
Prove or disprove: For any vector field $\boldsymbol{F}$, and smooth surfaces σ_1, σ_2, oriented upward, with the same boundary curve C, we have $\iint_{\sigma_1} \boldsymbol{F}\cdot\boldsymbol{n}\, dS = \iint_{\sigma_2} \boldsymbol{F}\cdot\boldsymbol{n}\, dS$.

False: Let $\boldsymbol{F}$ be an inverse square field, with positive constant, σ_1 be the top half of the sphere $x^2+y^2+z^2=1$, and σ_2 be the disk $x^2+y^2=1$, $z=0$.
For σ_1, the vector field is perpendicular to the surface at each point, thus $\iint_{\sigma_1} \boldsymbol{F}\cdot\boldsymbol{n}\, dS > 0$. For σ_2, the vector field is parallel to the surface at each point, and so $\iint_{\sigma_2} \boldsymbol{F}\cdot\boldsymbol{n}\, dS = 0$.