Quiz 19

16.6.8:

$$\begin{aligned}
\Phi &= \iint_R \langle x+y, y+z, z+x\rangle \cdot \langle 1,1,1\rangle \; dA \\
&= \iint_R 2(x+y+z)\, dA \\
&= 2\iint_R 1\, dA \\
&= 2\cdot\frac{1}{2}(1)(1) \\
&= 1.
\end{aligned}$$

16.7.6:

$$\begin{aligned}
\Phi &= \iiint_{x^2+y^2+z^2\le a^2} \operatorname{div}\left\langle z^3, -x^3, y^3\right\rangle \; dV \\
&= \iiint_{x^2+y^2+z^2\le a^2} (0+0+0)\, dV \\
&= \iiint_{x^2+y^2+z^2\le a^2} 0\, dV \\
&= 0.
\end{aligned}$$

HB:
Prove or disprove: If the flux of a vector field $\boldsymbol{F}$ through a closed, outward-oriented surface is positive, then the surface encloses a source.

True: Applying the Divergence Theorem, we see that $0 < \iint_\sigma \boldsymbol{F}\cdot\boldsymbol{n}\, dS = \iiint_G \operatorname{div}\boldsymbol{F}\, dV$. Since the triple integral is positive, the integrand must be positive somewhere inside of the region G. In other words, $\operatorname{div}\boldsymbol{F} > 0$ at some point, that is, a source, inside of G.