Quiz 18

16.3.24:
Note that the vector field, $\boldsymbol{F}$, is constant. So, $\boldsymbol{F} = \langle a, b \rangle$ for some real numbers a, b. Since $\nabla(ax + by) = \langle a, b \rangle = \boldsymbol{F}$, then $\boldsymbol{F}$ is conservative.

16.4.3:

$$\begin{aligned}
\oint_C 3xy\,dx + 2xy\,dy &= \int_{-2}^{4} \int_{1}^{2} (2y - 3x)\,dy\,dx \\
&= \int_{-2}^{4} \left[y^2 - 3xy\right]_1^2 \,dx \\
&= \int_{-2}^{4} 3 - 3x\,dx \\
&= \left[3x - \frac{3}{2}x^2\right]_{-2}^{4} \\
&= 12 - 24 + 6 + 6 \\
&= 0.
\end{aligned}$$

HB:
Prove or disprove: $\oint_C \frac{-y\,dx + x\,dy}{x^2 + y^2}$ is the same for any circle C, centered at the origin, oriented counterclockwise.

True: Parameterize C by $\boldsymbol{r}(t) = \langle a\cos t, a\sin t \rangle, \quad 0 \leq t \leq 2\pi$.

$$\begin{aligned}
\oint_C \frac{-y\,dx + x\,dy}{x^2 + y^2} &= \int_0^{2\pi} \frac{-a\sin t(-a\sin t) + a\cos t(a\cos t)}{a^2\cos^2 t + a^2\sin^2 t}\,dt \\
&= \int_0^{2\pi} \frac{a^2\sin^2 t + a^2\cos^2 t}{a^2\cos^2 t + a^2\sin^2 t}\,dt \\
&= \int_0^{2\pi} dt \\
&= 2\pi.
\end{aligned}$$