Quiz 17

16.2.21:

$$\begin{aligned}\int_C yz\,dx - xz\,dy + xy\,dz &= \int_0^1 e^{3t}e^{-2}(e^t) - e^te^{-t}(3e^{3t}) + e^te^{3t}(-e^{-t})\,dt \\ &= \int_0^1 -3e^{3t}\,dt \\ &= \left[-e^{3t}\right]_0^1 \\ &= 1 - e^3.\end{aligned}$$

16.3.20:
Note: $\nabla(x^2y + \sin y) = \boldsymbol{F}$, and so, the vector field is conservative.

$$\boldsymbol{r}(0) = \langle 0, 0\rangle, \quad \boldsymbol{r}(\pi) = \langle \pi, \frac{\pi}{2}\rangle$$

$$\int_C \boldsymbol{F}\cdot d\boldsymbol{r} = \pi^2\frac{\pi}{2} + 1 - 0 = \frac{\pi^3}{2} + 1.$$

HB:
Prove or disprove: If $\int_C \boldsymbol{F}\cdot d\boldsymbol{r} = 0$ for a simple closed curve C then $\boldsymbol{F}$ is conservative.

This is false. Two counterexamples are:

1. $\boldsymbol{F} = \langle -x\sin(\sqrt{x^2+y^2}), y\sin(\sqrt{x^2+y^2})\rangle$, C is the circle of radius π centered at the origin.
2. $\boldsymbol{F} = \begin{cases} \mathbf{0} & \text{if } \|\boldsymbol{r}\| = 1 \\ \langle 1, 0\rangle & \text{otherwise} \end{cases}$, C is the unit circle (or any circle of radius 1 centered at the origin if in 3-space).

In both of these cases, the vector field is not conservative, but is identically zero on the curve. So, the line integral is zero.