Quiz 16

13.2.50:

$$\boldsymbol{r}(t) = \langle e^{-2t}, \cos t, 3\sin t\rangle$$
$$\boldsymbol{r}(t) = \langle 1, 1, 0\rangle \Rightarrow e^{-2t} = 1 \Rightarrow t = 0$$
$$\boldsymbol{r}'(t) = \langle -2e^{-2t}, -\sin t, 3\cos t \Rightarrow \boldsymbol{r}'(0) = \langle -2, 0, 3\rangle$$
$$\boldsymbol{l}(t) = \langle 1-2t, 1, 3t\rangle \qquad x = 0 \Rightarrow t = \frac{1}{2} \Rightarrow \left(0, 1, \frac{3}{2}\right)$$

16.1.22:

$$\boldsymbol{F} = \langle e^{xz}, 3xe^{y}, -e^{yz}\rangle$$
$$\nabla \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \partial_x & \partial_y & \partial_z \\ e^{xy} & 3xe^{y} & -e^{yz} \end{vmatrix} = \langle -ze^{yz}, xe^{xz}, 3e^{y}\rangle$$
$$\nabla \cdot (\nabla \times \boldsymbol{F}) = 0 + 0 + 0 = 0.$$

HB: Evaluate $\int_C \sqrt{a^2 - x^2 - y^2}\, ds$ where C is the line segment $y = x$, $\quad -\frac{a}{\sqrt{2}} \leq x \leq \frac{a}{\sqrt{2}}$.
Hard Answer:

$$\int_C \sqrt{a^2 - x^2 - y^2}\, ds = \int_{-\frac{a}{\sqrt{2}}}^{\frac{a}{\sqrt{2}}} \sqrt{a^2 - 2x^2} \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2}\, dx$$
$$= a \int_{-\frac{a}{\sqrt{2}}}^{\frac{a}{\sqrt{2}}} \sqrt{1 - \left(\frac{\sqrt{2}}{\sqrt{a}} x\right)^2} \sqrt{1+1}\, dx$$
$$(x = \frac{a}{\sqrt{2}} \sin\theta) \Rightarrow = a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$
$$= \frac{a^2}{2} \left[1 + \cos(2\theta)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$
$$= \frac{a^2}{2} \left(\frac{\pi}{2} + (-1) - (-\frac{\pi}{2}) - (-1)\right)$$
$$= \frac{\pi a^2}{2}.$$

Easy Answer: Recall that a line integral can be expressed as the area above a curve and under a surface. In this case, we are finding the area under a sphere and above one of its diameters. In other words, the area of a semicircle. Thus, the solution is $\frac{\pi a^2}{2}$.