Quiz 15

13.2.44:

$$\boldsymbol{y}'(t) = \langle \cos t, \sin t \rangle \Rightarrow \boldsymbol{y}(t) = \langle \sin t, -\cos t \rangle + \boldsymbol{C}$$
$$\langle 1, -1 \rangle = \boldsymbol{y}(0) = \langle 0, -1 \rangle + \boldsymbol{C} \Rightarrow \boldsymbol{C} = \langle 1, 0 \rangle$$
$$\boldsymbol{y}(t) = \langle \sin t + 1, -\cos t \rangle.$$

13.3.24:

$$\boldsymbol{r}'(t) = \langle -3\cos^2 t \sin t, 3\sin^2 t \cos t \rangle$$

$$\begin{aligned} s &= \int_0^t \|\boldsymbol{r}'(u)\| \, du \\ &= \int_0^t \sqrt{9\cos^4 u \sin^2 u + 9\sin^4 u \cos^2 u} \, du \\ &= \int_0^t 3\cos u \sin u \sqrt{\cos^2 u + \sin^2 u} \, du \\ &= \int_0^t 3\cos u \sin u \, du \\ &= \left[\frac{3}{2} \sin^2 u \right]_0^t \\ &= \frac{3}{2} \sin^2 t. \end{aligned}$$

$$\sin t = \sqrt{\frac{2}{3}s} \Rightarrow \cos t = \sqrt{1 - \frac{2}{3}s}$$

$$\boldsymbol{r}(s) = \left\langle \left(1 - \frac{2}{3}s\right)^{\frac{3}{2}}, \left(\frac{2}{3}s\right)^{\frac{3}{2}} \right\rangle, \quad 0 \le s \le \frac{3}{2}.$$

HB:
If a runner's path is modeled by the vector valued function $\boldsymbol{r}(s) = \langle x(s), y(s) \rangle$, which is parameterized by a unit speed parametrization, what distance does the runner travel from $s = a$ to $s = b$, $a < b$?

Since the function is parameterized by a unit speed parametrization, ignoring units, the distance traveled equals the time spent traveling. Thus, the runner travels $b - a$ units.