Quiz 14

15.7.16:

$$\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^3 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = 2\pi \left[-\cos\phi\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\frac{1}{3}\rho^3\right]_0^3 = 9\pi\sqrt{2}.$$

15.8.22:

$$(x,y) = (2u, v) \qquad \frac{x^2}{4} + y^2 = 1 \qquad \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$\begin{aligned}
\iint\limits_{\frac{x^2}{4}+y^2\le 1} e^{-(x^2+4y^2)} \, dA &= \iint\limits_{u^2+v^2\le 1} e^{-(4u^2+4v^2)} 2 \, dA \\
&= \int_0^{2\pi}\int_0^1 2re^{-4r^2} \, dr \, d\theta \\
&= 4\pi \left[-\frac{1}{8}e^{-4r^2}\right]_0^1 \\
&= \frac{\pi}{2}(1-e^{-4})
\end{aligned}$$

HB:
Note that the integral will be positive if $36 - x^2 - 4y^2 - 9z^2 > 0$ throughout the solid G, and negative if $36 - x^2 - 4y^2 - 9z^2 < 0$ throughout the solid G. So, the integral $\iiint\limits_G 36 - x^2 - 4y^2 - 9z^2 \, dV$ will be maximized when every point satisfying $36 - x^2 - 4y^2 - 9z^2 \ge 0$ is in G, and G contains no other points. This inequality is equivalent to $\left(\frac{x}{6}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 \le 1$. Thus, G is the solid ellipsoid inside $\left(\frac{x}{6}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$.