

Quiz 13

15.5.9:

$$\begin{aligned}\iiint_G xy \sin(yz) \, dV &= \int_0^\pi \int_0^1 \int_0^{\frac{\pi}{6}} xy \sin(yz) \, dz \, dy \, dx \\&= \int_0^\pi \int_0^1 [-x \cos(yz)]_0^{\frac{\pi}{6}} \, dy \, dx \\&= \int_0^\pi \int_0^1 \left(x - x \cos\left(\frac{\pi}{6}y\right) \right) \, dy \, dx \\&= \int_0^\pi \left[xy - \frac{6}{\pi}x \sin\left(\frac{\pi}{6}y\right) \right]_0^1 \, dx \\&= \int_0^\pi \left(x - \frac{3}{\pi}x \right) \, dx \\&= \left(1 - \frac{3}{\pi} \right) \left[\frac{1}{2}x^2 \right]_0^\pi \\&= \frac{\pi - 3}{2}\pi.\end{aligned}$$

15.6.5:

$$\begin{aligned}M &= \int_0^1 \int_0^x dy \, dx = \int_0^1 dx = \left[\frac{1}{2}x \right]_0^1 = \frac{1}{2} \\ \bar{x} &= \frac{1}{1/2} \int_0^1 \int_0^x x \, dy \, dx = 2 \int_0^1 x^2 \, dx = \frac{2}{3}[x^3]_0^1 = \frac{2}{3} \\ \bar{y} &= 2 \int_0^1 \int_0^x y \, dy \, dx = \int_0^1 [xy]_0^x \, dx = \int_0^1 x^2 \, dx = \frac{1}{3}[x^3]_0^1 = \frac{1}{3}.\end{aligned}$$

HB: Note that the surfaces project onto $x^2 + y^2 = 4$ in the xy -plane.

$$\begin{aligned}\int_0^{2\pi} \int_0^2 \int_{2r^2}^{12-r^2} r \, dz \, dr \, d\theta &= 2\pi \int_0^2 (12r - 3r^3) \, dr \\&= 2\pi \left[6r^2 - \frac{3}{4}r^4 \right]_0^2 \\&= 2\pi(24 - 12) = 24\pi.\end{aligned}$$