Quiz 12

15.3.30:

$$\begin{aligned}\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy &= \int_0^{\frac{\pi}{2}} \int_0^1 r\cos(r^2)\,dr\,d\theta \\ &= \frac{\pi}{2}\left[\frac{1}{2}\sin(r^2)\right]_0^1 \\ &= \frac{\pi}{4}\sin(1)\end{aligned}$$

15.4.30:

$$1 = u^2, 4 = v^2 \Rightarrow u = \pm 1, v = \pm 2 \qquad u + v = 3 \Rightarrow u = 1, v = 2$$

$$\mathbf{r}(u,v) = \langle u^2, v^2, u+v\rangle \qquad \frac{\partial \mathbf{r}}{\partial u} = \langle 2u, 0, 1\rangle \qquad \frac{\partial \mathbf{r}}{\partial v} = \langle 0, 2v, 1\rangle$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2u & 0 & 1 \\ 0 & 2v & 1 \end{vmatrix} = \langle -2v, -2u, 4uv\rangle \longrightarrow \langle -4, -2, 8\rangle$$

Plugging in the point and normal vector into the equation for a plane gives us $-4x - 2y + 8z = 12$ or $2x + y - 4z + 6 = 0$.

HB:

$$f_x(x,y) = \frac{x}{\sqrt{x^2+y^2}} \qquad f_y(x,y) = \frac{y}{\sqrt{x^2+y^2}}$$

$$\begin{aligned}S &= \iint\limits_{x^2+y^2\le a^2} \sqrt{1 + \left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2}\,dA \\ &= \iint\limits_{x^2+y^2\le a^2} \sqrt{1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}}\,dA \\ &= \iint\limits_{x^2+y^2\le a^2} \sqrt{2}\,dA \\ &= \int_0^{2\pi}\int_0^a \sqrt{2} r\,dr\,d\theta \\ &= 2\pi\left[\frac{\sqrt{2}}{2}r^2\right]_0^a \\ &= \sqrt{2}a^2\pi\end{aligned}$$