1. $\int_{-\infty}^{\infty} e^{-x^2}\,dx =$

2. (a) Two identical pipes with radii R intersect at right angles. Find the volume of their intersection.

   (b) Three identical pipes with radii R intersect at the same point at right angles. Find the volume of their intersection.

3. For each of the following, give an example or explain thoroughly why no example exists.

   (a) A non-zero differentiable function of two variables defined everywhere that is integrable over the entire xy-plane.

   (b) A bounded differentiable function of two variables defined everywhere that is not integrable over the entire xy-plane.

   (c) A bounded differentiable function of two variables defined everywhere that is not integrable over the unit disk.

   (d) A differentiable function of two variables that is integrable over the unit disk but not the unit square.

   (e) A bounded function of two variables defined everywhere that is not integrable over the unit square.

   (f) A differentiable function of two variables defined everywhere that has just one critical point, and that the critical point is a local maximum but not a global maximum.

4. $\int_1^2 \int_{\sqrt{x}}^{x} \sin(\frac{\pi x}{2y})\,dy\,dx + \int_2^4 \int_{\sqrt{x}}^{2} \sin(\frac{\pi x}{2y})\,dy\,dx =$

5. By integrating $\int_0^1 \int_0^1 \frac{1}{1-xy}\,dx\,dy$ two different ways, show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

   Hint: Use a familiar series from Calc II for one way. Fir the other way, use a transformation $x = \frac{u-v}{\sqrt{2}}, y = \frac{u+v}{\sqrt{2}}$. Afterwards, applying the trg sub $u = \sqrt{2}\sin\theta$ and using the identities $\sin\theta = \cos(\frac{\pi}{2} - \theta), \cos\theta = 1 - 2\sin^2(\frac{\theta}{2})$ might be helpful.

6. Let $f : [0,1] \times [0,1] \longrightarrow \mathbb{R}$ be defined by $f(x,y) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 3y & \text{if } x \notin \mathbb{Q} \end{cases}$ Show that f is not integrable by showing that $\int_0^1 \int_0^1 f(x,y)\,dy\,dx$ cannot be evaluated.

7. Let $E = \{(x,y,z) : x^2 + y^2 + z^2 \geq 1\}$. Evaluate the improper integral $\iiint\limits_E \frac{dV}{(x^2+y^2+z^2)^2}$.

8. Find the smallest volume bounded by the coordinate planes (in the first octant) and a tangent plane (in the first octant) to the graph of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.