Exam 4 Review

Unless otherwise stated, all curves are oriented counterclockwise when viewed from above, and surfaces are oriented upward/outward.

1. $\int_C x\,ds, \quad C = \langle t, t^2 \rangle, \quad 0 \le t \le 2.$
2. $\int_C x \sin y\,dx + 3xyz\,dz, \quad \boldsymbol{r}(t) = \langle t, t^2, t^3 \rangle, \quad o \le t \le \sqrt{\pi}.$
3. $\int_C (x+y^2)\,dx + (1+x^2)\,dy$ where C is the boundary of the region enclosed by $y = x^2, \quad y = x^3$.
4. $\oint \left\langle x^2 y, \frac{1}{3}x^3, xy \right\rangle \cdot d\boldsymbol{r}$ where C is the intersection of $z = y^2 - x^2$ and $x^2 + y^2 = 1$.
5. $\iint_\sigma (y^2 + z^2)\,dS$ where σ is the portion of $x = 4 - y^2 - z^2$ with $x \ge 0$.
6. $\int_C (y^2 + 2xy)\,dx + (x^2 + 2xy)\,dy$ where C is the line segment from (-1,2) to (3,1).
7. $\iint_\sigma (\text{curl}\left\langle x \sin^2 z, 3x, z + \tan^{-1}(xy) \right\rangle) \cdot \boldsymbol{n}\,dS$, σ is the portion of $z = \sqrt{9 - x^2 - y^2}$ inside $x^2 + y^2 = 4$.
8. $\oint \langle xy, yz, zx \rangle \cdot d\boldsymbol{r}$ where C is the triangle (1,0,0), (0,1,0), (0,0,1).
9. $\int_C xy^4\,ds. \quad C: x^2 + y^2 = 9, \quad x \ge 0.$
10. $\iint_\sigma (\text{curl}\,\langle yz, xz, xy \rangle) \cdot \boldsymbol{n}\,dS$, σ is the portion of $z = 9 - x^2 + y^2$ above $z = 5$.
11. $\int_C e^y\,dx + 2xe^y\,dy$ where C is the unit square.
12. $\iint_\sigma yz\,dS$ where σ is the portion of the plane $x + y + z = 1$ in the first octant.
13. $\int_C 7xy\,dx + 5x^2y^3\,dy$ where C is $x = y^3$ from (1,1) to (0,0).
14. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's Theorem.
15. Find the area of the region enclosed by the curve defined by $\boldsymbol{r}(t) = \langle 2\cos^3 t, 2\sin^3 t \rangle, \quad 0 \le t \le 2\pi$.
16. In each example, find the flux of the vector field $\boldsymbol{F}$ through the surface σ:
    (a) $\boldsymbol{F} = \langle 3x, xz, z^2 \rangle$, σ is the surface of the region bounded by $z = 4 - x^2 - y^2$ and $z = 0$.
    (b) $\boldsymbol{F} = \langle x, y, z \rangle$, σ is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ (no bottom).
    (c) $\boldsymbol{F} = \langle x^2 + \sin(yz), y - xe^{-z}, z^2 \rangle$, σ is the surface of the region enclosed by $x^2 + y^2 = 4$, $x + z = 2$, $z = 0$.
    (d) $\boldsymbol{F} = \langle x, -y, 0 \rangle$, σ is the portion of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.
    (e) $\boldsymbol{F} = \langle 2, 5, 3 \rangle$, σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ inside $x^2 + y^2 = 1$.
17. Show that $\nabla \times (\nabla f) = \mathbf{0}$.
18. Show that $\nabla \cdot (\nabla \times \langle f, g, h \rangle) = 0$.

19. Decide whether each of the following statements are true or false. If true, justify your answer. If false, give a counterexample.

   (a) For every pair of differentiable functions f, g in one variable, $\int_C f(x)\,dx + g(y)\,dy$ is path independent.

   (b) $\boldsymbol{F} = \langle xy^2, x^2z \rangle$ is an example of a vector field.

   (c) If σ is a closed surface and $\boldsymbol{F}$ is any vector field, then the flux of $\boldsymbol{F}$ through σ is 0.

   (d) If curl$\boldsymbol{F} = \mathbf{0}$ with $\boldsymbol{F}$ a 3-dimensional vector field with continuous first order partial derivatives, then $\int_C \boldsymbol{F} \cdot d\boldsymbol{r} = 0$ for any simple closed curve C.

   (e) $\int_C \frac{x}{x^2+y^2}\,dx - \frac{y}{x^2+y^2}\,dy$ is path independent.