Calculus 3 – Spring 2012

Written Homework #5

Due 2/24/2012

Problem 1) A square metal plate with side length 2 centered at $(0,0)$ in the xy-plane has been heated and has a temprature

$$T(x,y) = 100 - 4x^2 - 2y^2$$

at the point (x,y). A bug is standing at the point $(-1/3, 5/8)$ and decides its feet are cold so it starts moving in the south-east direction at a rate of 3 units per hour. Calculate the rate of change in temprature that the bug feels when it begins moving.

Problem 2) Let $f(x,y,z) = x^2 + y^2 + z^2$ Find all points on the level surface $f(x,y,z) = 1$ so that the tangent plane to the surface at the point is parallel to both the vector $\vec{\mathbf{i}} - \vec{\mathbf{j}} + \vec{\mathbf{k}}$ and the vector $\vec{\mathbf{k}}$.

Problem 3) For any real number t define the vector $s(\vec{t}) = x(t)\vec{\mathbf{i}} + y(t)\vec{\mathbf{j}} + z(t)\vec{\mathbf{k}}$ and for a fixed value t_0 define the vector $s'(\vec{t_0}) = x'(t_0)\vec{\mathbf{i}} + y'(t_0)\vec{\mathbf{j}} + z'(t_0)\vec{\mathbf{k}}$. Suppose $s(\vec{t_0}) = (x_0, y_0, z_0)$, and $\|s'(t_0)\| = 1$. Let $F(x,y,z)$ be any differentiable function. Prove that the directional derivative $F_{s'(\vec{t_0})}(x_0,y_0,z_0)$ of F at (x_0,y_0,z_0) in the direction of $s'(\vec{t_0})$ is the same as the derivative $\frac{d}{dt}\left(F\left(x(t),y(t),z(t)\right)\right)|_{t=t_0}$. i.e. prove

$$F_{s'(\vec{t_0})}(x_0,y_0,z_0) = \frac{d}{dt}\left(F\left(x(t),y(t),z(t)\right)\right)|_{t=t_0}.$$

Problem 4) Let $f(x,y,z) = x\cos(y)\sin(z)$, $g(x,y,z) = x\sin(y)\sin(z)$, and $h(x,y,z) = x\cos(z)$ for $x > 0$, $0 \le y \le 2\pi$, and $0 \le z \le \pi$. Calculate

$$|\left(\nabla f \times \nabla g\right) \cdot \nabla h|.$$

Problem 5) For any positive integer p a function $f(x,y,z)$ is called homogeneous of order p if for any real number t the following equation holds:

$$f(tx,ty,tz) = t^p f(x,y,z).$$

Prove that if the function f is homogeneous of order p then

$$xf_x(x,y,z) + yf_y(x,y,z) + zf_z(x,y,z) = pf(x,y,z).$$