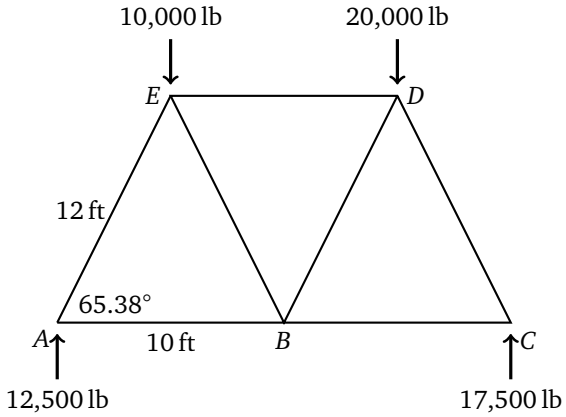
Calculus 3 – Spring 2012

Written Homework #3

Due 2/10/2012

Chapter 13

1) A Warren Truss is a structure for bearing a weight such as a roof or a bridge with two supports at either end of a gap. The truss in the figure below is loaded by weights at points D and E and is supported by vertical forces at points A and C. The horizontal bars in the truss are 10 ft long and the diagonal bars are 12 ft. Angles A and C are 65.38°. The arrows in the figure represent vertical forces in the given direction with the specified magnitude.



Each bar exerts a force at the two joints at its ends. The two force vectors are parallel to the bar, equal in magnitude, and opposite in direction. If the bar pushes on the joints at its ends, then the bar is under compression, and if it pulls it is under tension, and the magnitude of the force is called the magnitude of the tension or compression.

Engineers need to know the magnitude of the compression or tension in each of the bars of the truss to prevent them from bending or breaking. To determine these magnitudes, we use the fact that at each joint the sum of the external forces from the weights and supports and the pushing and pulling forces exerted by the bars is zero. Find the magnitudes for all seven bars in the following order:

(a) Joint A; Bars AB, AE

(b) Joint C; Bars BC, CD

(c) Joint D; Bars BD, DE

(d) Joint E; Bar BE

2) Provided $\vec{v}$ and $\vec{w}$ are not zero vectors, the following formula is true. Provide a conclusive algebraically valid argument as to why it must be true.

$$\frac{\vec{v}\cdot\vec{w}+||\vec{v}\times\vec{w}||}{||\vec{v}||\,||\vec{w}||}=\sin\theta-\sin\left(\theta-\frac{\pi}{2}\right).$$

3) A triangle has a side of length 5 and a side of length 6. The length of the remaining side is not known, but the triangle's area is known to be 12. What is the angle of the triangle where the sides of known length meet?

Chapter 14

4) Consider a class of m students and a year with n days. Let $q(m,n)$ denote the probability, expressed as a number between 0 and 1, that at least two students in the class have the same birthday. Surprisingly, $q(23,365)\approx 0.5073$. (This means that there is slightly better than a $1/2$ chance that at least two students in a class of 23 have the same birthday.) A general formula for q is complicated, but it can be shown that

$$\frac{\partial q}{\partial m}\approx+\frac{m}{n}(1-q)\qquad\text{and}\qquad\frac{\partial q}{\partial n}\approx-\frac{m^2}{2n^2}(1-q).$$

(These approximations hold when n is a good deal larger than m, and m is a good deal larger than 1.)

(a) Explain why the $+$ and $-$ signs in the approximations for $\frac{\partial q}{\partial m}$ and $\frac{\partial q}{\partial n}$ are to be expected.

(b) Suppose there are 21 students in a class. What is the approximate probability that at least two students in the class have the same birthday? (Assume that a year always has 365 days.)

(c) Suppose there is a class of 24 students and you know that no one was born in the first week of the year. (This has the effect of making $n=358$.) What is the approximate value of q for this class?

(d) If you want to bet that a certain class of 23 students has at least two matching birthdays, would you prefer to have more students added to the class or to be told that no one in the class was born in December?

5) The cost of producing one unit of a product is given by

$$c=a+bx+ky$$

where x is the amount of labor used (in man hours) and y is the amount of raw material used (by weight) and a, b, and k are constants. What does $\partial c/\partial x=b$ mean? What is the practical interpretation of b?

6) A company uses x hours of unskilled labor and y hours of skilled labor to produce $F(x,y)=60x^{2/3}y^{1/3}$ units of output. It currently employs 400 hours of unskilled labor and 50 hours of skilled labor. The company is planning to hire an additional 5 hours of skilled labor.

(a) Use a linear approximation to decide by about how much the company can reduce its use of unskilled labor and keep its output at current level.

(b) Calculate the exact value of the reduction.