Calculus 3 - Spring 2012
Written Homework #2
Due 2/3/2012

13.1: 1. Find all vectors $\vec{v}$ in the plane such that $\left\|\vec{v} - \hat{i}\right\| = \sqrt{5}$ and $\left\|\vec{v} + \hat{j}\right\| = \sqrt{5}$.

13.2: 2. A plane wants to fly due North while in 70 km/hr winds blowing to the Southeast. Assuming the plane can fly at 600 km/hr in calm weather, and that the plane is flying at maximal speed, what direction should the plane point? What will be its ground speed?

13.3: 3. (a) Prove the Cauchy-Schwarz Inequality: For any two n-dimensional vectors, we have that

$$|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \, \|\vec{v}\| .$$

When will this be an equality?

(b) Use the Cauchy-Schwarz Inequality to prove the Triangular Inequality: For any two n-dimensional vectors, we have that

$$\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$$

When will this be an equality?

13.3: 4. Suppose $\vec{u}$ and $\vec{v}$ are n-dimensional vectors such that $\vec{u} \cdot \vec{w} = \vec{v} \cdot \vec{w}$ for every n-dimensional vector $\vec{w}$. Show that $\vec{u} = \vec{v}$.

13.4: 5. Suppose $\vec{u}$ and $\vec{v}$ are 3-dimensional vectors such that $\vec{u} \times \vec{w} = \vec{v} \times \vec{w}$ for any 3-dimensional vector $\vec{w}$. Show that $\vec{u} = \vec{v}$.

13.4: 6. Show that for any 3-dimensional vectors $\vec{u}, \vec{v}$, we have that

$$|\vec{u} \cdot \vec{v}|^2 + \|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2 \, \|\vec{v}\|^2 .$$

13.4: 7. Let $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ and $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ be non-parallel, non-zero vectors lying in $z = mx + ny + c$. Let S be a parallelogram defined by $\vec{u}$ and $\vec{v}$, and R be the parallelogram that is the projection of S into the xy-plane.

(a) Find the area of S.

(b) Find the area of R.

(c) Show that

$$(\text{Area of } S) = \sqrt{m^2 + n^2 + 1} \cdot (\text{Area of } R).$$