Calculus 3 – Spring 2012

Written Homework #10

Due 4/13/2012

Problem 1) A particle begins traveling along the parametric curve

$$\vec{\mathbf{r}}(t) = t^2\vec{\mathbf{i}} + t^2\vec{\mathbf{j}} + t^3\vec{\mathbf{k}}$$

at time $t = 0$. find the exact value of time it takes for the particle to travel a distance of exactly one unit.

Problem 2) Suppose that the parametric curve $\vec{\mathbf{r}}(t) = x(t)\vec{\mathbf{i}} + y(t)\vec{\mathbf{j}} + v(t)\vec{\mathbf{k}}$ has constant speed, i.e. $\|\vec{\mathbf{v}}\| = c$ for some constant c. Prove that the velocity vector $\vec{\mathbf{v}}(t)$ is perpendicular to the acceleration vector $\vec{\mathbf{a}}(t)$. **Hint:** $\frac{d}{dt}\left(\frac{dx}{dt}^2\right) = 2\frac{dx}{dt}\frac{d^2x}{dt^2}$.

Problem 3) Let $\vec{\mathbf{F}}(x,y,z) = x\vec{\mathbf{i}} + (x+y)\vec{\mathbf{j}} + (x-y+z)\vec{\mathbf{k}}$. consider the line l given by $x(t) = 5+t$, $y(t) = 6-2t$, $z(t) = 7-3t$.

- Find a point where the vector field $\vec{\mathbf{F}}$ is parallel to the line l,
- Find a point where the vector field $\vec{\mathbf{F}}$ is perpendicular to the line l.

Problem 4) For any function $f(x,y,z)$ the gradient of f, grad$f(x,y,z)$ defines a vector field. Find a function $f(x,y,z)$ such that grad$f(x,y,z) = \frac{x}{\sqrt{x^2+y^2}}\vec{\mathbf{i}} + \frac{y}{\sqrt{x^2+Y^2}}\vec{\mathbf{j}} + \vec{\mathbf{k}}$.