Calculus 3 - Spring 2012
Written Homework #1
Due 1/27/2012

Chapter 12

Section 1

1. Using optimization from Calculus 1, find the point on the x-axis that is closest to the point (a, b, c). Find the distance between the two points.
2. Describe in words the region represented by $xyz = 0$.

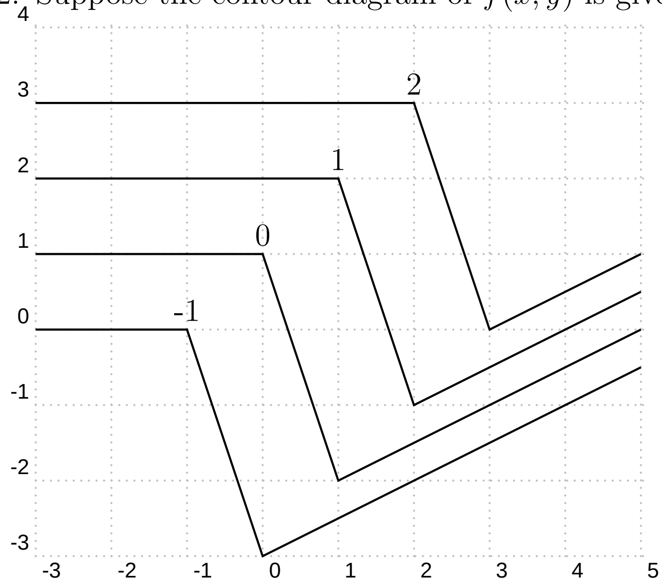
Section 2

1. Give a formula for a function whose graph is a cone with circular cross section, vertex at (2,-1,5), and opening in the negative x direction. (The axis of symmetry is parallel to the x-axis.)
2. By setting one variable constant find a plane that intersects the graph of $z^2 = x^3y^2 - x + 1$ in a
   (a) parabola.
   (b) pair of intersecting lines.
   (c) pair of parallel lines.

Section 3

1. (a) Give an example of a continuous surface whose level curves are circles centered on the z-axis and whose radii, with increasing z, are
      i. constant.
      ii. decreasing at a constant rate.
      iii. increasing at an increasing rate.
   (b) Give an example of a continuous surface whose level curves are circles centered on the z-axis and
      i. whose radii are 1 for even z and 2 for odd z.
      ii. whose z values are 1 for even r and -1 for odd r, where r is the radius of the circular level curve.

2. Suppose the contour diagram of $f(x, y)$ is given by



Sketch the contour diagrams of

(a) $f(x-1, y)$

(b) $2f(x, y)$

(c) $f(x, -y) + 3$

(d) $f(y, x)$

Section 4

1. Find an equation of the plane that contains the line $y = -2x + 3$ in the plane $z = 5$, and goes through the point $(2, -7, 23)$.

2. We have that the slope of a line in two dimensions is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{distance traveled in dependent variable}}{\text{distance traveled in independent variable}} = \frac{\Delta y}{\Delta x}.$$

We can define the slope of a line in three dimensions as

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{distance traveled in dependent variable}}{\text{distance traveled in independent variables}} = \frac{\Delta z}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}.$$

If we envision standing at a point on the plane and looking in any direction, we would be looking along a line. We can now find the slope these lines.

Compute the following slopes using the plane $f(x, y) = ax + by + c, \ (a, b > 0):$

(a) Find the slope of f in the direction $\Delta x > 0, \ \Delta y = 0$.

(b) Find the slope of f in the direction $\Delta x = 0, \ \Delta y > 0$.

(c) For $0 \leq \theta \leq 2\pi$, find the slope of f in the direction $\Delta x = \cos\theta, \ \Delta y = \sin\theta$.

(d) Find the direction that maximizes the slope, and the value of the maximal slope.

(e) Find the direction that minimizes the slope, and the value of the minimal slope.

(f) Find all directions where the slope is 0.

Section 5

1. Let $F(x, y, z) = \ln(x^2 - 6x + y^2 + 2y + z^2 + 10)$.
   (a) Find the domain of F.
   (b) Describe in words the level surface $F(x, y, z) = 2$. Be as specific as you can.

2. Given a point p, define $d(p)$ as the distance from p to the origin. Define $a_y(p)$ as the (shortest) distance from p to the y-axis. Find an equation for the surface made up of all points p where the ratio $\dfrac{d(p)}{a_y(p)}$ is 2. Identify this surface.

Section 6

1. Find a function $y = g(x)$ so that

$$f(x,y) = \begin{cases} \dfrac{5x^6 - 5x^4y + x^2y^2 - y^3}{x^2 - y} & \text{if } y \neq x^2 \\ g(x) & \text{if } y = x^2 \end{cases}$$

   is continuous everywhere.

2. Show that

$$\lim_{(x,y)\to(0,0)} \frac{5xy^2}{16x^2 + 9y^4}$$

   does not exist.

3. Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy(x - y)}{x^2 + y^2}$$

   exists.