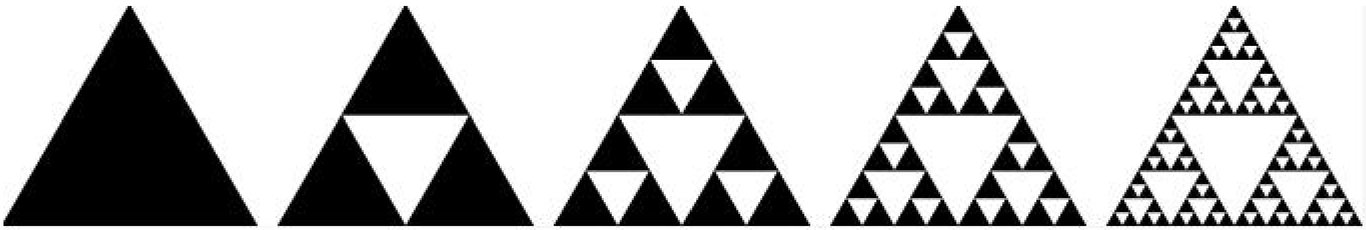
MATH 2300-002
Due Monday, March 8

1. The *Sierpinski Gasket* is defined to be the "limit" of the following sequence:



Starting with an equilateral triangle with side length 1, the first term in the sequence, remove the "upside down" equilateral triangle that has its vertices on the edges of the big triangle. (Note: This would be a triangle with side lengths of 1/2). This leaves us with three equilateral triangles, each with side lengths of 1/2, which is the second term in the sequence. The next step is to remove triangles from each of the three smaller triangles, thus giving 9 equilateral triangles, each with side length of 1/4, which is the third term in the sequence. This process is then repeated indefinitely to get the limit.

   (a) Find the total area ("black space") of the nth term of the sequence. Use this to find the area of the Sierpinski Gasket.

   (b) Find the perimeter (total length of edges separating the "black space" and "white space" as shown in the figure) of the nth term of the sequence. Use this to find the perimeter of the Sierpinski Gasket.

2. The **expected value** of an event is defined to be the sum of the probability of each outcome occurring times the value, or payoff, of that outcome. The expected value tells you, on the average, how much you will gain or lose each time the event happens.

For example, you pay a dollar to play a game. You then flip a coin twice. If the first flip is tails, you win \$1. If the first flip is heads, and the second flip is tails, you win \$2. If both flips are heads, you win \$4. Assuming the coin is fair (P(heads) = P(tails) = 1/2) the expected value of your winnings would be:

$$\left(\frac{1}{2}\right)\cdot(1)+\left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot(2)+\left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot(4)=\frac{1}{2}+\frac{1}{2}+1=2$$

(To find these probabilities, I used the fact that flipping heads n times in a row is $(1/2)^n$, and flipping heads n times in a row, and then a tails is $(1/2)^n \cdot (1/2) = (1/2)^{n+1}$)

This shows that, on the average, you will win \$2 each time you play. Since you pay \$1 each time, you would make a profit of \$1, on average, per play. If you instead payed \$2 each time, you would, on average, neither win nor lose any money. In this case, the game is called **fair**.

(a) A game is played where you repeatedly flip a fair coin until you get a tails, or until you have flipped the coin N times, whichever comes first. (Note, the possible results are T, HT, HHT, HHHT, ... , HHH...HHT, HHH...HHH) For flipping k heads, you win $\$2^k$. How much should you pay for this game to be fair? *Note: It might help to do this for N=1, 2, 3, and then for arbitrary N.*

(b) Suppose the game stopped after 8 flips (N=8), and it costs $4 to play. Would you play?

(c) The same game is played, except that there is no limit as to the number of flips. That is, you keep flipping until you get a tails. How much should you pay to make this game fair?

(d) Suppose the game costs $10 to play. Would you play? (Think Carefully)

3. Two trains 100 miles apart are moving toward each other; each one is going at a speed of 10 miles per hour. A fly starting on the front of one of them flies back and forth between them at a rate of 40 miles per hour. It does this until the trains collide and crush the fly. What is the total distance the fly has flown?

   (a) Solve the problem using infinite series. *Hint:* $d = rt$

   (b) Solve the problem using another method.

   (*Hint*: The website `http://www.math.utah.edu/~cherk/mathjokes.html` might be helpful.)