1. (20) Evaluate the following integrals:

(i)
$$\int \sin^2 \theta \cos^3 \theta \ d\theta;$$

(ii)
$$\int_0^e \ln x \, dx;$$

(iii)
$$\int_0^\infty \frac{4}{x^2 + 16} \, \mathrm{d}x;$$

(iv)
$$\int \sin(\ln x) \, dx$$
.

- 2. (20)
 - (i) Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (y+1)^2 \sin x, \quad y\left(\frac{\pi}{2}\right) = 0.$$

(ii) Verify that $y(x) = \sum_{k=0}^{\infty} \frac{x^{3k}}{(3k)!}$ is a solution to the differential equation y''' - y = 0.

- 3. (10) Starting with the geometric series $\sum_{k=0}^{\infty} x^k$:
- (i) find the sum of the series $\sum_{k=1}^{\infty} kx^{k-1}$ for |x| < 1, and

(ii) find the sum of the series $\sum_{k=1}^{\infty} kx^k$ for |x| < 1.

- 4. (15)
- (i) Does the sequence $\left\{\frac{n\sin\left(\frac{\pi n}{2}\right)}{2n^2+1}\right\}_{n=1}^{\infty}$ converge or diverge?

(ii) Does the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left(\frac{\pi}{4}\right)^{2k+1}$ converge or diverge? If it converges, find its sum.

5. (10) Determine if the following series diverge, converge conditionally, or converge absolutely.

(i)
$$\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k^2}$$
;

(ii) $\sum_{k=1}^{\infty} \frac{(-1)^k (k^2 + 1)}{2k^2 + k - 1}.$

6. (10) Find the interval of convergence for $\sum_{k=2}^{\infty} \frac{(-1)^k}{k} (2x+3)^k.$

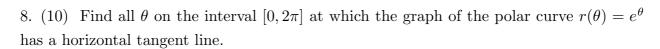
- 7. (20)
- (i) Evaluate the definite integral $\int_0^{\pi/2} x \cos x \, dx$ using integration by parts.

(ii) Find the Maclaurin series for $f(x) = x \cos x$ by any method.

(iii) Express the definite integral $\int_0^{\pi/2} x \cos x \, dx$ as the sum of an infinite series.

(iv) What does the series you found in part (iii) converge to?

Math 2300	page 9 of 9
	1 - 0



9. (10) Find the area of the region enclosed by the curve $r = a \sin \theta$, where a > 0 is a constant.

Name: _			
Section			

University of Colorado

Mathematics 2300: Final Exam

May 7, 2008

No calculators, formula sheets, notes or books are allowed.

Justify your answers. Correct answers with no justification may not receive full credit.

Problem	Points	Score
1	20	
2	20	
3	10	
4	15	
5	10	
6	10	
7	20	
8	10	
9	10	
Total	125	