1. (20) Evaluate the following integrals:

(i) $\int \sin^2\theta \cos^3\theta \, \mathrm{d}\theta$;

(ii) $\int_0^e \ln x \, \mathrm{d}x$;

(iii) $\displaystyle\int_0^\infty \frac{4}{x^2+16}\,\mathrm{d}x;$

(iv) $\displaystyle\int \sin(\ln x)\,\mathrm{d}x.$

2. (20)

(i) Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (y+1)^2 \sin x, \quad y\left(\frac{\pi}{2}\right) = 0.$$

(ii) Verify that $y(x) = \sum_{k=0}^{\infty} \frac{x^{3k}}{(3k)!}$ is a solution to the differential equation $y''' - y = 0$.

3. (10) Starting with the geometric series $\sum_{k=0}^{\infty} x^k$:

(i) find the sum of the series $\sum_{k=1}^{\infty} kx^{k-1}$ for $|x| < 1$, and

(ii) find the sum of the series $\sum_{k=1}^{\infty} kx^k$ for $|x| < 1$.

4. (15)

(i) Does the sequence $\left\{ \dfrac{n \sin\left(\frac{\pi n}{2}\right)}{2n^2+1} \right\}_{n=1}^{\infty}$ converge or diverge?

(ii) Does the series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left(\frac{\pi}{4}\right)^{2k+1}$ converge or diverge? If it converges, find its sum.

5. (10) Determine if the following series diverge, converge conditionally, or converge absolutely.

(i) $\displaystyle\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k^2}$;

(ii) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k(k^2+1)}{2k^2+k-1}$.

6. (10) Find the interval of convergence for $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}(2x+3)^k$.

7. (20)

(i) Evaluate the definite integral $\int_0^{\pi/2} x \cos x \, \mathrm{d}x$ using integration by parts.

(ii) Find the Maclaurin series for $f(x) = x\cos x$ by any method.

(iii) Express the definite integral $\displaystyle\int_0^{\pi/2} x\cos x \, \mathrm{d}x$ as the sum of an infinite series.

(iv) What does the series you found in part (iii) converge to?

8. (10) Find all θ on the interval $[0, 2\pi]$ at which the graph of the polar curve $r(\theta) = e^{\theta}$ has a horizontal tangent line.

9. (10) Find the area of the region enclosed by the curve $r = a \sin\theta$, where $a > 0$ is a constant.

Name: _____

Section: _____

University of Colorado

Mathematics 2300: Final Exam

May 7, 2008

No calculators, formula sheets, notes or books are allowed.
Justify your answers. Correct answers with no justification may not receive full credit.

Problem	Points	Score
1	20	
2	20	
3	10	
4	15	
5	10	
6	10	
7	20	
8	10	
9	10	
Total	125	