1. (10)

Given that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$ and that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$, calculate the values of

(i) $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k^2}$;

(ii) $\displaystyle\sum_{k=1}^{\infty} \frac{3-k^2}{k^4}$.

2. (30) Determine whether each of the following series converges absolutely, converges conditionally or diverges.

(i) $\displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{k}{k+\ln k}\right)$;

(ii) $\displaystyle\sum_{k=1}^{\infty} \frac{5}{4+e^k}$;

(iii) $\displaystyle\sum_{k=1}^{\infty} \frac{k\cos(k\pi)}{k^2+1}$.

3. (20) Determine whether each of the following series converges absolutely, converges conditionally or diverges.

(i) $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$;

(ii) $\displaystyle\sum_{k=1}^{\infty} \frac{5+(-1)^k}{\sqrt{k}}$.

4. (20) Find the interval of convergence for each of the following series.

(i) $\displaystyle\sum_{k=0}^{\infty} k!(x-5)^k;$

(ii) $\displaystyle\sum_{k=0}^{\infty} (-1)^{k+1}\frac{x^k}{k}.$

5. (10) Find the Taylor series for $e^x$ about $x = 1$.

6. (10)

(i) Find the interval of convergence for the series $\sum_{k=0}^{\infty} \left( \frac{x-1}{3} \right)^k$.

(ii) What function does this series converge to?

**Name:** ________________________________

**Section:** ________________________________

University of Colorado

Mathematics 2300: Third Midterm Exam

April 9, 2008

*No calculators, formula sheets, notes or books are allowed.*
*Justify your answers. Correct answers with no justification may not receive full credit.*

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 30 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 10 | |
| 6 | 10 | |
| Total | 100 | |