

1. (10) Evaluate the indefinite integral $\int \frac{3x - 4}{x^2 - 3x + 2} \, dx$.

2. (15) Find a **nonzero** number b such that $\int_0^b \ln(x) \, dx = 0$.

3. (25)

- (i) Find the solution $y(x)$ of the differential equation $(1 + x^4) \frac{dy}{dx} = \frac{x^3}{y^2}$ that satisfies $y(0) = 2$.

- (ii) Find the general solution of the differential equation $y' + y = \cos(e^x)$.

4. (15)

(i) Verify that $\sinh x$ is a solution to the differential equation $y'' - y = 0$.

(ii) Solve the initial value problem $y'' - 5y' = 0$, where $y(0) = 3$ and $y'(0) = 10$.

(iii) Find the general solution of the differential equation $y'' + 2y' + 10y = 0$.

5. (10) Determine if the sequence $\left\{ \frac{n \cos n}{n^2 + 1} \right\}_{n=1}^{\infty}$ converges, and if it does, find its limit.

6. (10)

(i) Starting with $n = 1$, find the general term of the sequence

$$\frac{3}{2^2 - 1^2}, \quad \frac{4}{3^2 - 2^2}, \quad \frac{5}{4^2 - 3^2}, \dots$$

(ii) Determine whether the sequence above converges, and if so, find its limit.

7. (15)

(i) Show that the sequence $\left\{ \frac{n!}{12^n} \right\}_{n=1}^{\infty}$ is eventually monotone.

(ii) Show that the sequence $\{\ln(n)\}_{n=1}^{\infty}$ is eventually monotone.

(iii) Give an example of a divergent monotone sequence.

Name: _____

Section: _____

University of Colorado

Mathematics 2300: Second Midterm Exam

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No calculators, formula sheets, notes or books are allowed.

Justify your answers. Correct answers with no justification may not receive full credit.

Problem	Points	Score
1	10	
2	15	
3	25	
4	15	
5	10	
6	10	
7	15	
Total	100	