Math 2400 Midterm Review 3

1. Compute the integral $\int_R xy\ dA$ over the region R bounded by the curves $x = 1$, $x = -1$, $x = y^2$, $y = -1$, and $y = 1 + x^2$.

2. A swimming pool is circular with a $40ft$ diameter. The depth is constant along east-west lines and increases linearly from $2ft$ at the south end to $7ft$ at the north end. Find the volume of water the pool can hold.

3. Find the volume of the solid determined by the inequalities $y \leq z^2 + x^2$ and $x^2 + y^2 + z^2 \leq 3$.

4. Evaluate the integral $\int_R x^2 e^{x^2}/y^2\ dA$, where R is the region bounded by $y = 1/x$, $y = 2/x$, $y = x$, and $y = 2x$. *Hint:* Try a change of variables.

5. Let $\vec{F}(x, y) = -x^2\vec{i} + 2xy\vec{j}$. Find an equation for the curve that goes through the point $(1, 2)$ and is perpendicular to $\vec{F}$ at every point. *Hint:* $dy/dx = y'(t)/x'(t)$.

6. Show that the flowlines of $\vec{F}(x, y) = \vec{i} + x\vec{j}$ are parabolas. Graph the flowlines of $\vec{F}$ through the points $(-2, 2)$, $(-2, 0)$, and $(1, 1)$.

7. Parametrize the following surfaces:

   (a) The portion of the surface $z = x + 3$ inside the cylinder $x^2 + y^2 = 7$.

   (b) The portion of the sphere of radius 1 centered at the origin that is inside the sphere of radius 1 centered at $(0, 0, 1)$.

8. Compute the line integral of $\vec{F}(x, y, z) = ay\vec{i} + bx\vec{j} - cz\vec{k}$ along the curve from the origin to the point $(1, 2, 3)$. Find conditions on a, b, and c so that the FTCLI applies.

9. A $80kg$ man carries a $10kg$ can of paint up a helical staircase that encircles a silo with a radius of $20m$. If the silo is $90m$ high and the man makes exactly three complete revolutions in 6 minutes, how much work is done by the man against gravity in climbing to the top. *Hint:* The force of gravity on an object is $\vec{F} = -mg\vec{k}$ where m is the mass of the object and $g \approx 9.81m/s^2$ is the acceleration due to gravity.

10. Suppose that $\vec{F}$ is a continuous nonzero vector field defined on all of 2-space. Let C be the unit circle centered at the origin and oriented counter clockwise. Suppose that $\int_C \vec{F} \cdot d\vec{r} = 0$. Show that F is tangent to C at atleast two points.