1

step:	5	2	5	6	5	1	3	2	4	3	4
	$[\varphi$	$\to$	$(\varphi$	$\to$	$\psi)]$	$\to$	$[\theta$	$\to$	$(\varphi$	$\to$	$\psi)]$
value:	1	1	1	0	0	0	1	0	1	0	0

The first implication is contradictory. So the statement is a tautology.

2 (a): Let f be the assignment which takes only the value 1. We prove by complete induction on i that for every $i < m$, $\varphi[f] = 1$. Assume that this is true for all $j < i$. There are two cases.

Case 1. φ_i is S_k for some k. Then $\varphi_i[f] = f(k) = 1$.

Case 2. There are $j, k < i$ such that φ_i is $\varphi_j \wedge \varphi_k$. By the inductive hypothesis, $\varphi_j[f] = 1$ and $\varphi_k[f] = 1$. Hence $\varphi_i[f] = 1$.

This completes the inductive proof.

(b): There is a $\wedge$-construction sequence $\varphi_0, \ldots, \varphi_{m-1}$ such that ψ is φ_k for some k. Then by (a), $\psi[f] = \varphi_k[f] = 1$.

(c): Suppose that there is a $\wedge$-formula ψ such that $\neg S_0 \leftrightarrow \psi$ is a tautology. Let f be the sentential assignment which takes only the value 1. Then $(\neg S_0)[f] = 1 - S_0[f] = 0$ while $\psi[f] = 1$ by (b). This is a contradiction.

3 Recalling that $\exists v_1$ is $\neg\forall v_1\neg$, we get

$$\langle 4, 5, 2, 11, 8, 5, 1, 4, 10, 1, 3, 9, 10, 10, 5\rangle$$

4

$$[v_0 < v_1 \wedge \exists v_2[v_0 < v_2 \wedge v_2 < v_1 \wedge \forall v_3[v_0 < v_3 \wedge v_3 < v_1 \to v_3 = v_2]]]$$
$$\vee\ [v_1 < v_0 \wedge \exists v_2[v_1 < v_2 \wedge v_2 < v_0 \wedge \forall v_3[v_1 < v_3 \wedge v_3 < v_0 \to v_3 = v_2]]]$$

5

$$\exists v_1 \exists v_2[v_1 < v_2 \wedge v_2 < v_0 \wedge \forall v_3[v_3 < v_0 \to v_3 = v_1 \vee v_3 = v_2]]$$