Review for second midterm

1. Show that the set $\{\neg(v_0 = v_1)\}$ is inconsistent.

2. Give an example, with full details, of a formula φ and a variable v_j which occurs in φ, such that $\not\vdash \varphi \leftrightarrow \mathrm{Subb}^{v_i}_{v_j}\varphi$. Recall here Theorem 3.25, which has a misprint: one should have v_j instead of ψ_j.

3. Prove that if v_i does not occur free in φ, then $\vdash (\varphi \to \exists v_i \psi) \leftrightarrow \exists v_i(\varphi \to \psi)$.

4. Find a formula in prenex normal form equivalent to the formula

$$\forall v_0(v_0 < v_1 \to \exists v_1(v_1 < v_0)).$$

5. Let φ be the formula $v_0 = v_1 \wedge \forall v_2(v_2 = v_0 \to v_1 = v_3)$.
   (a) Indicate which occurrences of variables are free, and which ones bound.
   (b) Write out $\mathrm{Subf}^{v_0}_{v_1+v_2}\varphi$.
   (c) Write out $\mathrm{Subb}^{v_2}_{v_1}\varphi$.
   (d) Write out $\mathrm{Subb}^{v_2}_{v_1}\mathrm{Subf}^{v_0}_{v_1+v_2}\varphi$.
   (e) Write out $\mathrm{Subf}^{v_0}_{v_1+v_2}\mathrm{Subb}^{v_2}_{v_1}\varphi$.

6. In the language for $(\omega, <)$ give a formula φ such that for any assignment $a : \omega \to \omega$, $(\omega, <) \models \varphi[a]$ iff $a_0 < a_1 < a_1$ and there is an element between a_0 and a_1, but none between a_1 and a_2.

7. In the language for $(\omega, <)$, show that the following formula is universally valid:

$$\exists v_0[v_0 = v_1 \wedge \exists v_1(v_0 < v_1)] \to \forall v_0[v_0 = v_1 \to \exists v_1(v_0 < v_1)].$$