Math 6010 - Assignment 12

These problems are meant to introduce problems that are complete for various complexity classes. Less formal proofs suffice.

(1) The *Quantified Boolean Formula Problem* QBF asks whether a given quantified sentence

$$\exists x_1 \; \forall x_2 \; \exists x_3 \; \forall x_4 \ldots \quad \Phi(x_1, \ldots, x_n)$$

with Φ in $\wedge, \vee$ and $'$ is true. Hence QBF (sometimes also called TQBF or QSAT) generalizes SAT.

Argue that QBF is in PSPACE. Actually QBF is typical PSPACE-complete problem.

(2) Argue that deciding the following is in EXPTIME:

Input: code $\sharp(M)$ for a DTM M, $n \in \mathbb{N}$ in binary

Question: Does M halt on empty input in $\leq n$ steps?

Actually whether a DTM halts in $\leq n$ steps is a typical EXPTIME-complete problem.

Hint: Measure the complexity in $\log n$. The size of $\sharp(M)$ can be neglected.

(3) Argue that the previous problem for n given in unary is in P.

Actually this is a typical problem that is complete in P for NC-reductions (intuitively "probably not parallelizable").