

Math 6010 - Assignment 11

Due April 19, 2019

- (1) Show $NL \neq PSPACE$.
- (2) Show $PSPACE \neq EXPSPACE$ and $P \neq EXPTIME$.
- (3) (Space bound implies time bound) Let $s(n)$ be space constructible. Let M be a (non-deterministic or deterministic) TM with input and working tape that uses $\leq s(n)$ space on its working tape on any input of length n but which may not hold on all computational branches.

Show that there exists a TM N with input and working tape that uses $O(s(n))$ space on its working tape and halts in time $2^{O(s(n))}$ on all computational branches on any input of length n such that

$$L(M) = L(N).$$

Hint: Add a counter and an appropriate upper bound for time steps to M .

- (4) A Boolean formula $\Phi(x_1, \dots, x_n)$ is in *k-conjunctive normal form* (*k*-CNF) if

$$\Phi(x_1, \dots, x_n) = \bigwedge_{i=1}^{\ell} (a_{i1} \vee \dots \vee a_{ik})$$

where all $a_{ij} \in \{x_1, \dots, x_n, x'_1, \dots, x'_n\}$ are either arguments or negations of arguments.

k-SAT is the problem of deciding satisfiability of a Boolean formula in *k*-CNF.

Show that 2-SAT is in coNL (the complement of NL).

Hint: Start with assigning a truth value to one variable. What does this imply for the assignment of other variables?