Math 6010 - Assignment 9

Due March 29, 2021

(1) Prove the **Relativized Complementation Theorem**:
$B \leq_T A$ iff B and $\bar{B}$ are computably enumerable in A.

(2) Show that $\deg(A \oplus B)$ is the least upper bound for $\deg(A)$ and $\deg(B)$.

(3) Let $A \subseteq \mathbb{N}$. Show
$$A <_T A'$$
(i.e. A is Turing reducible to its jump $A' = \{x \mid \varphi_x^A(x) \downarrow\}$ but not conversely).

(4) Let $A, B \subseteq \mathbb{N}$. Prove
$$B \leq_T A \text{ iff } B' \leq_m A'.$$
Hint: For the "if"-direction show that B and $\bar{B}$ are A-computably enumerable using the Relativized Parameter and Enumeration Theorems.