Math 6010 - Assignment 7

Due March 8, 2021

(1) Prove that a partial function is recursive iff its graph is recursively enumerable.

(2) For $n \in \mathbb{N}$ let $\Delta^0_n := \Sigma^0_n \cap \Pi^0_n$. Show that

$$\Sigma^0_n \cup \Pi^0_n \subset \Delta^0_{n+1}$$

for $n \geq 1$. What about $n = 0$?

Hint: To show that the containment is proper let P be Σ^0_n but not Π^0_n and consider

$$Q(x,z) := (P(x) \wedge z = 0) \vee (\neg P(x) \wedge z = 1).$$

(3,4) Let $W_e :=$dom $\varphi^{(1)}_e$.

$$\begin{aligned} K &:= \{x \mid x \in W_x\} \\ H &:= \{x \mid 0 \in W_x\} \\ T &:= \{x \mid W_x = \mathbb{N}\} \\ F &:= \{x \mid W_x \text{ is finite}\} \end{aligned}$$

Show that K, H, T, F are all complete for appropriate levels in the arithmetical hierarchy (for those that were not covered in class yet).

(5) Which of the sets K, H, T, F and their complements are many-one reducible to each other?